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A novel robust fuzzy control of an uncertain system

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Abstract

This paper presents a robust control method combining the conventional proportional–integral–derivative (PID) scheme and the sliding mode fuzzy control scheme for a second-order non-linear system having uncertainties in the system dynamics. The goal of the proposed scheme is to force the response of the uncertain plant to follow that of the nominal model. The first phase of the design approach is to obtain a nominal PID controller for the nominal plant model. The poor performance of the sole PID scheme on the uncertain non-linear system motivates the proposal of the technique discussed here. To compensate for the deficiencies in the unit step response of the uncertain system, a fuzzy compensation scheme based on sliding mode control (SMC) is proposed and the PID loop is augmented by the proposed approach. It is shown that the performance with the proposed scheme is better than the sole PID-based control system. With the proposed technique, the response of the uncertain system converges to of the nominal system with admissible controller outputs. Furthermore, simulation results show that the proposed method produces consistent results even with noisy measurements.

Keywords

Fuzzy logic control, PID scheme, sliding mode control, sliding mode fuzzy control scheme

Introduction

Robust control has been one of the most interesting areas of control engineering during the last 40 years and until now has successfully been used in many engineering applications. It still maintains its popularity due to the increasing complexity of contemporary system models, and due to the continuous demand for precision and robustness. A robust control system provides desirable results in the presence of structural and parametric uncertainties. Determining controller structure and adjusting the parameters of the controller constitute two fundamental issues of designing a robust control system to obtain an acceptable system performance.

Robust control system design for non-linear systems with uncertainty is one of the most challenging tasks in control design methods because most of the processes have challenging non-linearities and uncertainties. There are a number of robust control methods in the literature, among which a prominent robust control scheme is fuzzy logic control (FLC), which was first proposed by Mamdani (1974), using the fuzzy sets theory of Zadeh (1965). FLC is based on human experience, which means FLC is an expert knowledge-oriented and rule-based control algorithm in contrast to classical transfer function-based control schemes. It consists of a set of rules that are described linguistically and the source of a descriptive rule base is an expert human. Because of this advantage, FLC has been used in many industrial applications as a control algorithm exploiting the knowledge of the expert.

The proportional–integral–derivative (PID) controller is another design alternative that may yield a robust closed loop if the gains are tuned well. It is widely used in industrial applications because of its advantages such as robust performance, simple structure, low cost in manufacturing and easy understanding in principle. However, the PID control method has some drawbacks in the control systems that are subject to uncertainty and imprecision. The reason for this is the coefficients for P, I and D actions are tuned for a nominal model and there may be severe deviations from the nominal response. In Aström and Hägglund (1995) and Papadopoulos (2014), PID control theory fundamentals and PID tuning issues are elaborated and an overview of the advances and applications in PID control has been presented in Vilanova and Visioli (2012). Recent studies in the literature demonstrate that the popularity of PID control is in progress for all fields and it should be emphasized that there are more than 100 industrial patents focusing on PID and its parameter tuning issues, indicating the continuous interest to PID scheme.

Another robust control scheme, SMC, which is a discontinuous non-linear control approach, was postulated in the early 1950s by Emelyanov, and evolved from the pioneering works cited in Itkis (1976), Utkin (1977, 1992) and Edwards and Spurgeon (1998). The purpose of the approach is to

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optimize the controller to obtain an acceptable closed-loop performance in the presence of uncertainties. To accomplish this, the design engineer needs to tackle two fundamental phases of the system response, namely the reaching mode and the sliding mode. The most important advantage of SMC is its robustness feature against uncertainties, external disturbances and variations in the parameters. However, a remarkable disadvantage of SMC is the chattering phenomenon caused by noisy measurements, limited sampling rate and switching delay in practice. The chattering is the high-frequency oscillating component of the control signal. To overcome this phenomenon, a common way is to introduce a boundary layer around the sliding hypersurface (Chen et al., 2014; Slotine and Li, 1991). Introducing a boundary layer gives concessions to the performance. An undesired result is the steady-state error. As a remedy, some researchers combined fuzzy logic systems and SMC to obtain a better SMC performance. As a result, two novel control structures emerged, namely fuzzy sliding mode control (FSMC) (Barrero, 2002; Bouarroudj et al., 2015; Mohammadi and Nafar, 2013; Özkop et al., 2015; Piltan et al., 2013; Roopaei et al., 2009; Saghafinia et al., 2014; Wong, 2001) and sliding mode fuzzy control (SMFC) (Lhee, 2001; Tu et al., 2000; Wai et al., 2002). The former is an adaptive control algorithm, and is used to form the equivalent control of SMC for unknown system dynamics identified by FLC. The latter is FLC based on SMC rules. There are numerous research outcomes about SMC in the literature, and it is still a state-of-the-art topic in the field of robust control (Ahmad and Zhu, 2015).

In addition to the aforementioned robust control schemes, blending of the techniques can also be considered among the robust control schemes. PID and SMFC combination is an example of this (Piltan et al., 2011), which propose a PID FLC method with a minimum rule base and the method is combined with SMC to adjust the gain updating factor and the slope of the sliding line. Results are discussed on a three-degrees-of-freedom robotic manipulator.

It is possible to extend the list of research outcomes that utilize the powerful aspects of SMC, fuzzy control and PID schemes. In spite of the alternatives available in the literature, the structure of the control system, the role of fuzzy inference mechanism and the availability of adaptation mechanism make the designs different from each other. To our best knowledge, this paper considers a novel combination of the mentioned approaches to obtain a stable and good performance closed-loop response.

The contribution of this paper is the combination of SMC and FLC, in such a way that the nominal controller is first obtained and is kept active in the closed loop. With this setup, the FLC is responsible for the removal of performance degradations utilizing the robustness properties of SMC scheme. This paper advances the subject area by introducing an approach that adjusts the magnitude of the SMC gain, which is a critical design parameter significantly influencing the overall performance. Results support the theoretical claims, as discussed throughout the paper.

This paper is organized as follows: the second section describes the mathematical model of the system under control. Then, classical PID controller design for nominal plant is explained. In the fourth section, classical SMC design and the proposed method are elaborated, and stability proof is provided. The fifth section is devoted to simulations and discussions, and then we provide a critical discussion considering the recent works. Finally, in the last section, concluding remarks are presented.

The dynamic model of the plant

The second-order uncertain non-linear system under control, which is studied in this paper, is described as in (1)–(2).

$$\begin{bmatrix} \dot{x}_{1p} \\ \dot{x}_{2p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} \begin{bmatrix} x_{1p} \\ x_{2p} \end{bmatrix} + \begin{bmatrix} 0 \\ D_1(.,.) \end{bmatrix} + \begin{bmatrix} 0 \\ c + D_2(.,.) \end{bmatrix} u_p \tag{1}$$

$$y_p = x_{1p} \tag{2}$$

where x_{1p} and x_{2p} are the state variables, and the variable u_p and y_p are the control input and the output of the uncertain model, respectively.a, b, c are the known process parameters that embody the nominal model of the system. In (1), D_1 and D_2 are the bounded uncertainties with $|D_1(x_{1p}, x_{2p})| < B_f$ and $0 < D_2(x_{1p}, x_{2p}) < B_g < c$.

PID control scheme for nominal plant

The nominal model, which is the known linear part of the uncertain system in (1)–(2), can be given as in (3)–(4). Figure 1 shows the block diagram of the nominal control system including the nominal process model denoted by $G_n(s)$. The controller acting on the process is a classical PID controller and for the approach presented in this paper, its gains are tuned once and kept constant throughout the runtime. The designer's assumption is that the closed loop in Figure 1 is stable, and the response of the system shown in Figure 1 will be considered the reference signal when the design advances to the control of the uncertain process, which is to be discussed. Stability of the nominal control loop is not a stringent demand, as there are many alternatives available in the framework of classical designs. The nominal model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c \end{bmatrix} u \tag{3}$$

$$y_n = x_1 \tag{4}$$

where x_1 and x_2 are the state variables, and the variable u and y_n are the input and the output of the nominal model, respectively. Assuming zero initial conditions, the transfer function

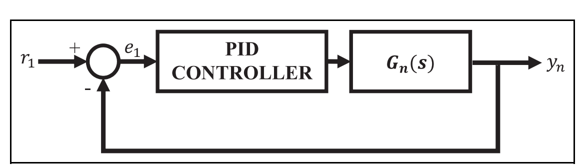

Figure 1. Proportional–integral–derivative (PID) control structure for the nominal system.

of the nominal model is obtained as given in (5), where we have certain assumptions, i.e. $c > 0$, $a, b \neq 0$, and nominal model is stable.

$$G_n(s) = \frac{c}{s^2 + bs + a} \tag{5}$$

During the design phase, we will choose reference signals such that $\dot{r}_1 = r_2$, where r_2 is the desired velocity profile. Further to this, the position and velocity errors are defined as $e_1 := r_1 - x_1$ and $e_2 := r_2 - x_2$, respectively. Using the error term e_1, the ideal PID controller can be given as in (6), where $E_1(s)$ is the Laplace transform of $e_1(t)$:

$$u_{PID}(s) = \left(k_p + \frac{k_i}{s} + k_d s\right)E_1(s) \tag{6}$$

where k_p, k_i and k_d are coefficients of the PID controller. Once determined, the coefficients of the PID controller will be assumed constant.

Classical SMC and proposed method

Defining σ as the switching variable, the objective of the SMC scheme is to push the error vector toward the sliding hypersurface denoted by the loci $\sigma(e_m, \dot{e}_m) = 0$, which is a particular subspace of the phase space that is attractive. The trajectories trapped to it are driven toward the origin of the phase space and the system states follow the reference trajectories. To accomplish this, the designer must handle two important phases of the SMC design, namely the deployment of the sliding hypersurface, governed by the switching function, and the selection of a control law, which makes the selected sliding subspace attractive. For second-order systems, the switching subspace is a line, a plane for third-order systems and a hypersurface for systems of order four or more. The plant considered here is a second-order one and the aforementioned switching variable is as described as

$$\sigma := \dot{e}_m + \lambda e_m, \ \lambda > 0 \tag{7}$$

where λ is positive slope parameter and $e_m := y_n - y_p$ is the model-following error. If for some time, say t_h, $\sigma(t_h) = 0$, then the solution of the differential equation in (7) is $e_m(t) = e_m(t_h)\exp(-\lambda(t - t_h))$ for $t \geq t_h$. This is valid if and only if the loci characterized by $\sigma = 0$ is a global attractor of the space spanned by the variables e_m and its time derivative. This is ensured by introducing a corrective term that uses the sign of the switching variable, σ, defined as $u_{SMC} := M\mathrm{sgn}(\sigma)$. In this representation $M > 0$ is a gain yet to be determined. The practice of the SMC scheme has shown that the choice above is quite vulnerable to noise and the smoothing scheme described below is adopted in most cases.

$$u_{SMC} := M\mathrm{sgn}(\sigma) \approx \frac{\sigma}{|\sigma| + \varepsilon}, \varepsilon > 0 \tag{8}$$

The approximation above removes the chattering effect to some extent while introducing a significant robustness against uncertainties. Here, ε is a parameter determining the slope around $\sigma = 0$. When $\varepsilon = 0$, the above function becomes the original sign function, whereas larger values of ε soften the sign function and a smooth transition from the origin is obtained. In the sequel, we will discuss how the signal in (8) joins the proposed solution in detail.

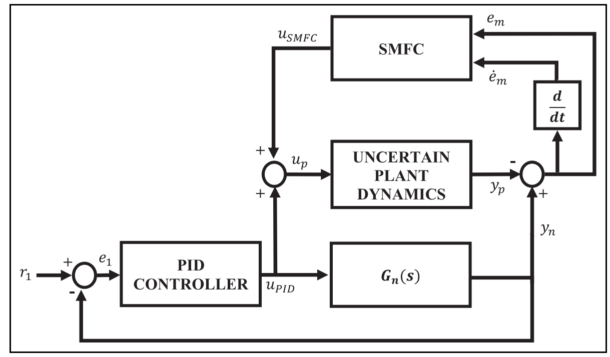

Figure 2. Architecture of the proposed control system.

In Figure 2, the structure of the proposed control system is shown. The bottom part of the diagram is a standard unity feedback loop containing the nominal plant model and the PID controller, yielding the desired stability and performance results on the nominal model. The uncertain system, which is under control, is installed on the overall mechanism as also shown in the figure. The input to the uncertain system is a combination of the nominal control signal and a correction term, which is derived using the model-following error, defined as the difference between the nominal model output and the uncertain plant output. The discontinuous component of the control signal is computed by the block named SMFC, which runs a fuzzy inference mechanism to synthesize a gain multiplying the discontinuous term. Having these in mind, the control signal is stated as in (9), where u_{PID} and u_{SMFC} are the components shown in Figure 2.

$$u_p = u_{PID} + u_{SMFC} \tag{9}$$

The second term above, which is used to guarantee the robustness of the controller, does not interfere in the control of the nominal system and a conservative control signal is avoided. While computing the control signal of the SMC part, a prime challenge in (8) is the selection of the SMC gain parameter denoted by M. As the value of M gets larger, the locus characterized by $\sigma = 0$ becomes more attractive and the robustness of the closed-loop system is improved. However, in such a case, high-frequency dynamics are provoked and the control signal is dominated by highly fluctuating terms that require costly actuation periphery. On the other hand, when the value of M is kept small, high-frequency components in the control signal disappear, yet the robustness property of the closed loop is lost to a certain extent. To offer a remedy to this problem, in this section, a SMC magnitude parameter (M) selection mechanism based on fuzzy logic is proposed.

Figure 3 illustrates the general structure of a fuzzy inference system. The designed fuzzy logic selection mechanism has two inputs and one output. The model-following error (e_m) and its derivative are the inputs to the fuzzy logic system,

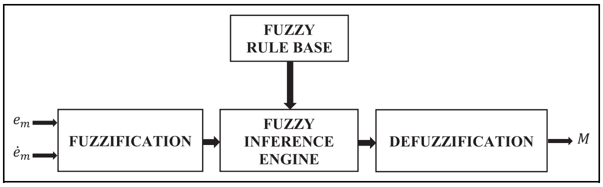

Figure 3. Structure of a fuzzy logic system.

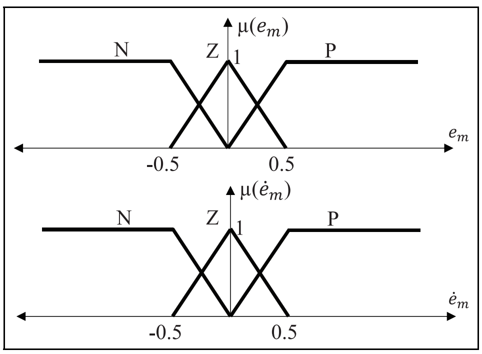

Figure 4. Selected membership functions along e_m and $\dot{e}_m$.

and the SMC magnitude parameter M is the output. Membership functions used in this work are triangular membership functions and for each input, three linguistic labels are employed to distinguish Negative (N), Zero (Z) and Positive (P) labels. These are shown in Figure 4 and Table 1 lists the fuzzy rule base. The consequent sets corresponding to Very Small (VS), Small (S), Large (L) and Very Large (VL) are determined based on the deployment of the error vector according to the origin and the switching subspace. Properly implemented, the fuzzy logic system will produce the gain of the signum function and the loop will be as shown in Figure 2. A critical question here is: does there exist such a value of M that ensures the attractiveness of the loci $\sigma = 0$? The following theorem proves this.

Theorem: *Let the system under control be as defined in (1)–(2) and the uncertainties in the system dynamics are bounded and they satisfy the inequalities* $\left|D_1\left(x_{1p}, x_{2p}\right)\right| < B_f$ *and* $0 < D_2\left(x_{1p}, x_{2p}\right) < B_g < c$. *Let the PID controller in Figure 2 meet the desired performance results when it operates only on the nominal plant model given in (3)–(4). The switching manifold defined by* $\sigma = 0$ *is a global attractor if the gain M in (8) satisfies the inequality* $M > \frac{|-ae_m + (\lambda - b)\dot{e}_m| + B_f + B_g|u_{PID}|}{c - B_g}$.

Proof. Choose the positive definite Lyapunov function candidate as

$$V = V_1(e_1, e_2) + \frac{1}{2}\sigma^2 \tag{10}$$

where V_1 is another Lyapunov function that is for the loop containing the nominal plant model and the PID controller pair. As the PID controller stabilizes this loop, without a proof, we assume that the time derivative of V_1 is negative.

Table 1. Fuzzy logic rule base.

$\dot{e}_m$	e_m		
	N	Z	P
N	VL	L	S
Z	L	VS	L
P	S	L	VL

N, Negative; Z, Zero; P, Positive; VA, Very Small; S, Small; L, Large; VL, Very Large.

With this in mind, differentiating (10) with respect to time yields

$$\dot{V} = \dot{V}_1(e_1, e_2) + \dot{\sigma}\sigma \tag{11}$$

The question now reduces to whether $\dot{\sigma}\sigma < 0$. According to (7), the time derivative of σ can be computed as below.

$$\dot{\sigma} = \ddot{e}_m + \lambda\dot{e}_m \tag{12}$$

As $e_m := y_n - y_p$ using the nominal system dynamics in (3)–(4) and the uncertain system dynamics in (1)–(2), we obtain the following equality.

$$\dot{\sigma} = (-ae_m + (\lambda - b)\dot{e}_m - D_1 - D_2u_{PID}) - (c + D_2)M\,\mathrm{sgn}(\sigma) \tag{13}$$

To obtain a Lyapunov stable closed loop, $\dot{\sigma}\sigma < 0$ must be ensured. To achieve that, we multiply both sides of (13) by σ. This will make us perform the following manipulations.

$$\begin{aligned} \dot{\sigma}\sigma &:= (-ae_m + (\lambda - b)\dot{e}_m - D_1 - D_2u_{PID})\sigma \\ &\quad - (c + D_2)M|\sigma| \\ &< |-ae_m + (\lambda - b)\dot{e}_m - D_1 - D_2u_{PID}||\sigma| \\ &\quad - (c - D_2)M|\sigma| \\ &\leq (|-ae_m + (\lambda - b)\dot{e}_m| + |D_1| + D_2|u_{PID}|)|\sigma| \\ &\quad - (c - D_2)M|\sigma| \\ &< \left(|-ae_m + (\lambda - b)\dot{e}_m| + B_f + B_g|u_{PID}|\right)|\sigma| \\ &\quad - (c - D_2)M|\sigma| \end{aligned} \tag{14}$$

For $\dot{\sigma}\sigma < 0$, the gain must satisfy the inequality in (15).

$$M > \frac{|-ae_m + (\lambda - b)\dot{e}_m| + B_f + B_g|u_{PID}|}{c - B_g} \tag{15}$$

The choice above ensures $\dot{V} = \dot{V}_1(e_1, \dot{e}_1) + \dot{\sigma}\sigma < 0$ and the mechanism depicted in Figure 2 becomes a stable control system.

Remark. Looking at (15), one sees that the lower bound is dependent on $|u_{PID}|$, which could be unbounded theoretically. In our work, we consider a band limited differentiator, which modifies the original derivative action to $\frac{k_d s}{1 + \tau s}$, where the inverse of τ determines the bandwidth of the valid differentiation.

The inequality in (15) is a guide to determine the rule base of the fuzzy system. Principally, the surface formed by the fuzzy system becomes higher when the $(e_m, \dot{e}_m)$ pair is in the first or the third quadrants of the phase space. Alternatively, the synthesized value is small when the $(e_m, \dot{e}_m)$ pair is in the second or the fourth quadrants. This choice is because of the deployment of the switching subspace. If the pair $(e_m, \dot{e}_m)$ is close to the $\sigma = 0$ subspace, then there is no need to choose a large M value. Alternatively, the points away from $\sigma = 0$ loci require a stronger attraction force and M is large in those points. An exception of this strategy is the near origin activity. As the ultimate goal is to force the pair $(e_m, \dot{e}_m)$ toward the origin, the fuzzy system produces the minimum values of M for the near origin subspace. This simply produces a bowl-shaped structure around the origin that connects smoothly to the aforementioned levels.

The fuzzy inference system employed in this work has the input–output relation given in (16).

$$M = \frac{\sum_{j=1}^{9} y_j \mu_{j1}(e_m)\mu_{j2}(\dot{e}_m)}{\sum_{j=1}^{9} \mu_{j1}(e_m)\mu_{j2}(\dot{e}_m)} \quad (16)$$

In this representation, $\mu_{j1}(e_m)$ is the membership function quantifying the membership value of e_m in the jth rule, whereas $\mu_{j2}(\dot{e}_m)$ is the membership function quantifying the membership value of $\dot{e}_m$ in the same rule.

The rule consequences are singletons denoted by y_is. The fuzzy model adopts the product inference rule and weighted average defuzzifier to obtain the crisp output. It would of course be possible to enhance the resolution of the proposed mechanism, yet the goal of our paper is to postulate the simplest and best performing structure. Therefore, a rule base containing nine rules is sufficient to obtain the prescribed properties over the four quadrants of the phase space and around the origin. In the next section, a set of simulations are discussed on an exemplar plant model.

Simulation studies

During the simulations, the nominal model of plant has the following parameters: $a = 5$, $b = 4$ and $c = 25$. The nominal control loop is as shown in Figure 1, and the PID controller parameters have been determined utilizing the Ziegler–Nichols tuning law. A further manual modification has been done to obtain the best possible response, which is the minimum overshoot and minimum rise time response. This has yielded $k_p = 8.55$, $k_i = 9.22$ and $k_d = 1.84$. In order not to produce unbounded control signals for discontinuous reference signals, a modified derivative action was used. More explicitly, the differentiation has been implemented as $\frac{k_d s}{1 + s/57.54}$. A good closed-loop performance for these selections has been observed in the simulations. The full model of the plant has the following bounded uncertainty terms.

$$D_1 := \frac{100 sin(x_{1p}) cos(x_{2p})}{1.01 + x_{1p}^2} < 100 = B_f \quad (17)$$

$$D_2 := 10 \frac{1 + 0.99 sin(x_{1p} x_{2p})}{1 + x_{1p}^2} < 20 = B_g \quad (18)$$

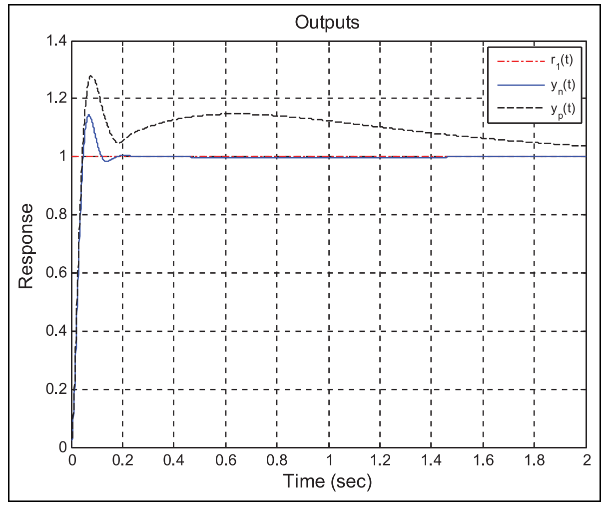

Figure 5. Comparison of the unit step responses of the nominal system and the uncertain system with only proportional–integral–derivative (PID) controller.

If the uncertain plant model is used in the control loop shown in Figure 1, the results seen in Figure 5 are obtained. For a better comparison, the initial conditions are assumed zero. The dashed curve is the response of the uncertain system with the sole PID controller. Though tuned using standard approaches, the PID controller is unable to compensate for the deficiencies caused by the uncertainty terms. This point is the motivating fact of this work and we introduce a mechanism as shown in Figure 2, where a sliding mode-based fuzzy compensation is added. The singletons in the consequent part of the fuzzy inference system are $y_{VS} = 10$, $y_S = 15$, $y_L = 25$ and $y_{VL} = 28$. The value predicted by the inequality in (15) is considered the lower bound for the defuzzifier parameters given above. The fuzzy system generates a M value, which multiplies the sgn(σ) term, where $\lambda = 12$ is the slope of the switching line and $\varepsilon = 0.2$ is the smoothing parameter for the sign function, chosen for the simulation studies. Smoothing of the sign function is a remedy to eliminate chattering, yet it introduces a boundary layer around the switching subspace. Larger values of ε introduce thicker boundary layers with supposedly poor performances, yet smaller values produce a better approximation to the original sign function that provokes the high-frequency components in the control signal. With the membership functions depicted in Figure 4, the prediction surface formed by the fuzzy system is shown in Figure 6, where the left subplot is a 3D view and the right one depicts how the switching subspace is located over the four quadrants of the phase space with the surface height coloured as shown along the colour bar.

In Figure 7, the response of the system shown in Figure 2 is shown together with the nominal plant response obtained from Figure 1. The initial output of the nominal model is zero whereas that of the uncertain system is chosen as -1, which is large enough to see the performance of the proposed scheme. It is seen that the proposed technique forces the uncertain system response to that of the nominal control loop. Clearly, the goal is achieved with a slight modification to the original PID control scheme.

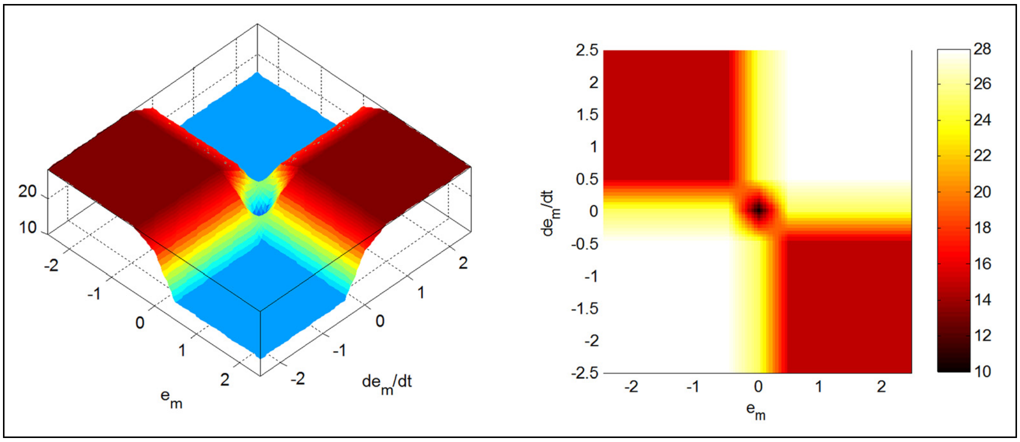

Figure 6. The value of M according to e_m and $\dot{e}_m$. Left: 3D plot, right: 2D coloured plot.

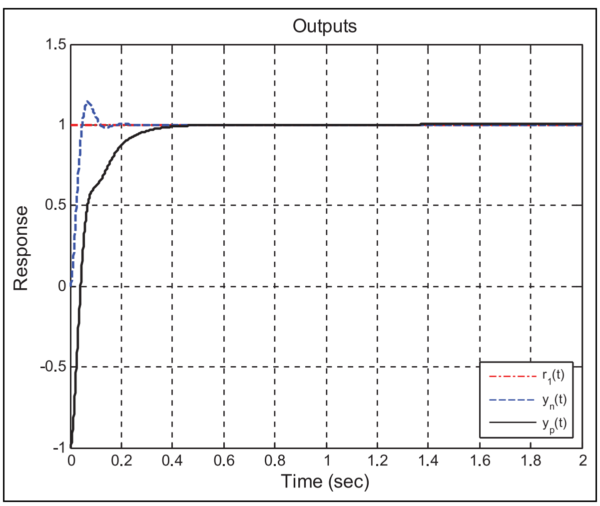

Figure 7. A comparison of the unit step responses of the nominal system and the uncertain plant with the proposed method.

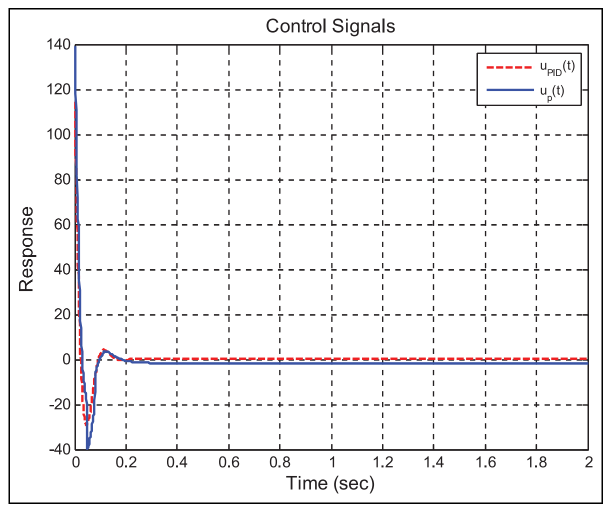

Figure 8. Nominal control signal (u_{PID}) and the control signal produced by the proposed approach (u_p).

Figure 8 shows the control signals produced for the nominal control system and the proposed scheme. The two signals look similar, yet the proposed technique synthesizes the details that handle the adverse effects of the uncertainty terms. The steady-state behaviours are also different, as seen from Figure 8.

In Figure 9, behaviour of the SMC magnitude gain term is illustrated. The early phase of the simulation produces a fast change in the value of M and a gradually converging regime is observed. The steady-state value of M is slightly larger than 10.

The trajectory followed in the phase space is depicted in Figure 10, where it is seen that the sliding regime emerges and the errors converge at the origin, as prescribed by the dynamical properties of the switching manifold and sign smoothing mechanism.

Finally, simulations have been repeated in presence of zero mean measurement noise from the interval $[-0.01, 0.01]$ with a probability very close to unity. It is observed that the plant with proposed method has a good tracking performance under noisy observations. However, when the control signal of the uncertain plant with proposed method (u_p) in Figure 11 is considered, the chattering problem that is a characteristic problem of SMC is observed as high-frequency fluctuations in the control signal. The chattering can be decreased by softening the sign function with the larger ε values, yet at this time, the system moves away from the advantageous domain of the SMC. The phase space behaviour with the noisy measurements is shown in Figure 12, where the chattering during the sliding regime is visible.

The simulations have been repeated for a continuous reference signal. The observations are noisy and the results are shown in Figures 13–16. In Figure 13, the response of the nominal system is shown together with the response of the uncertain system. Again, the initial conditions have been taken to be zero for a better comparison. The controller is the

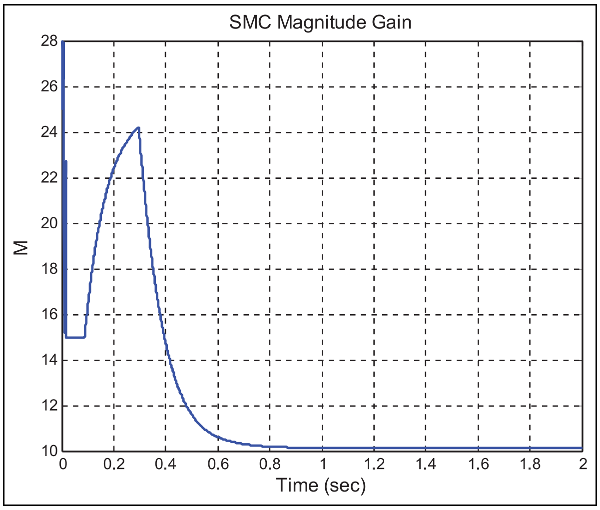

Figure 9. Sliding mode control (SMC) magnitude gain (*M*).

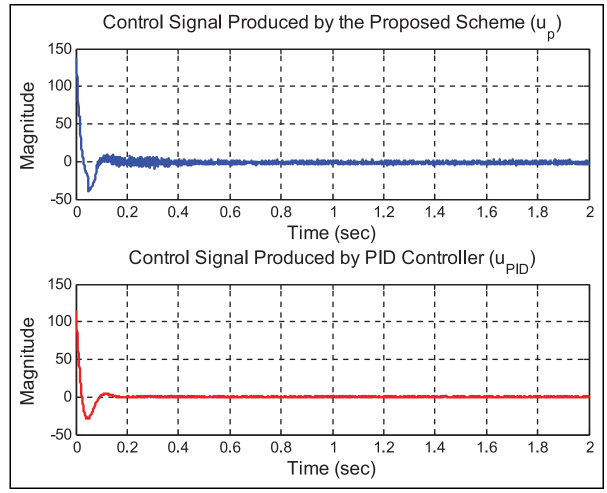

Figure 11. Comparison of the control signals of the nominal system and the plant with proposed method in presence of the noise.

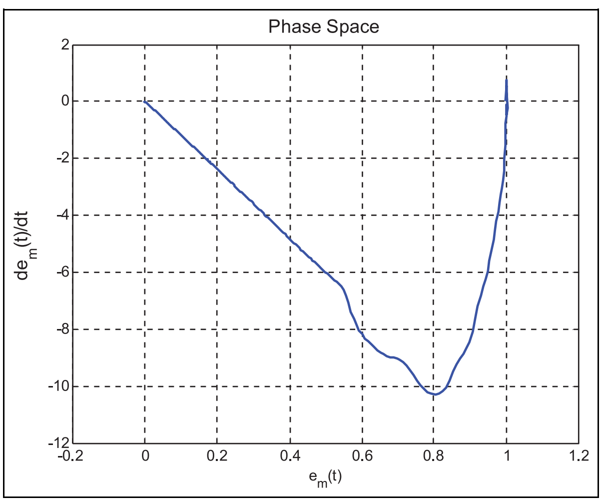

Figure 10. Phase space behaviour for the step reference.

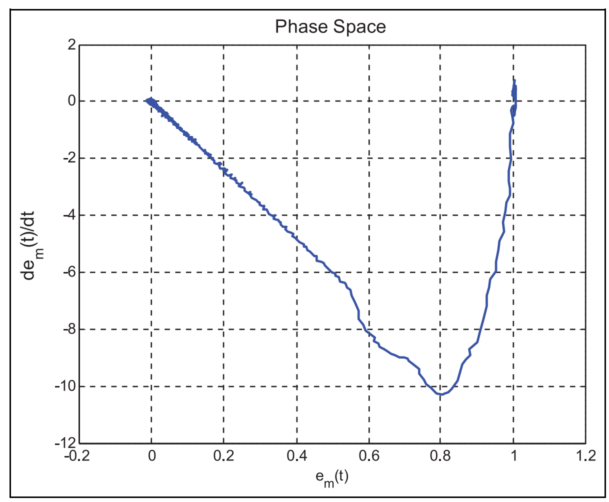

Figure 12. Phase space behaviour with noisy measurements. The reference signal is a step function.

sole PID controller and the reference signal is a sinusoidal with frequency 1 Hz. The solid curve is the uncertain system response and the performance is poor, as we desire it to follow the nominal system response. When the proposed scheme is executed, the results change to those shown in Figure 14, where it is seen that the uncertain system follows the output of the nominal model precisely. The initial output of the uncertain system for this case is now set to -1 to see how it approaches the nominal system output. The phase space behaviour is illustrated in Figure 15, and the produced control signals for these conditions are shown in Figure 16. Clearly, the presence of noise affects the smoothness of the control signal adversely and provokes the chattering, yet the sliding mode is successfully maintained around the switching subspace, as shown in Figure 17.

According to the simulation results, while the closed-loop performance without the proposed method is unsatisfactory, the proposed method is able to eliminate disadvantages caused by the uncertainties. Furthermore, it is observed that the proposed method produces consistent results even in presence of the noise. An exemplar case with two different reference signals is discussed through a set of simulations and it is concluded that the PID control loops can be enhanced by slight modifications and a good closed-loop performance and improved robustness against uncertainties can be achieved simultaneously.

A critical discussion

In this section, a number of recently published works focusing on FSMC and PID control combination are considered for comparison. Esfahani and Azimirad (2013) and Esfahani et al. (2014) focused on a feedback loop where the reference tracking error is used by a PID controller, whose output is manipulated as the switching variable. This requires a series

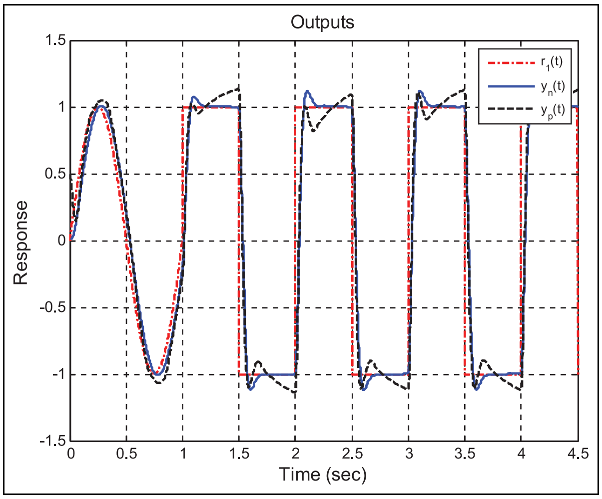

Figure 13. Responses of the nominal system and the uncertain system with only proportional–integral–derivative (PID) controller.

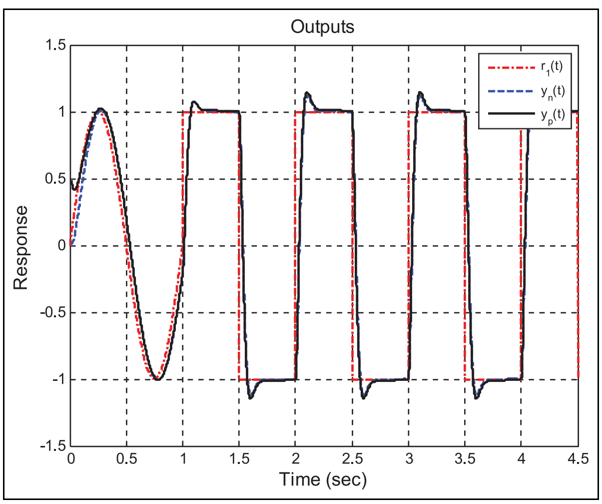

Figure 14. Responses of the nominal system and the uncertain plant with the proposed method.

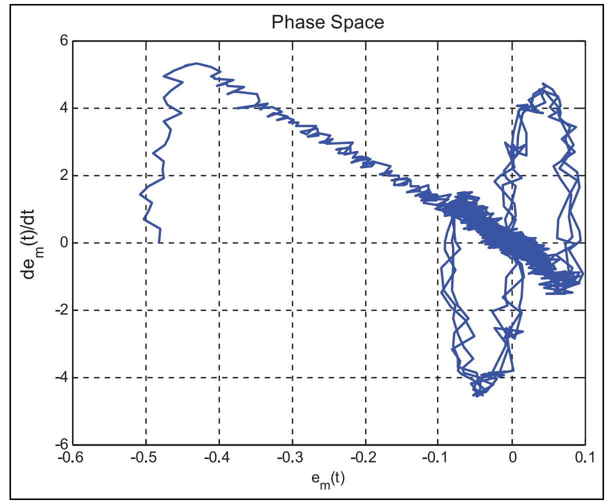

Figure 15. Phase space behaviour with the chosen reference signal. The observations are noisy.

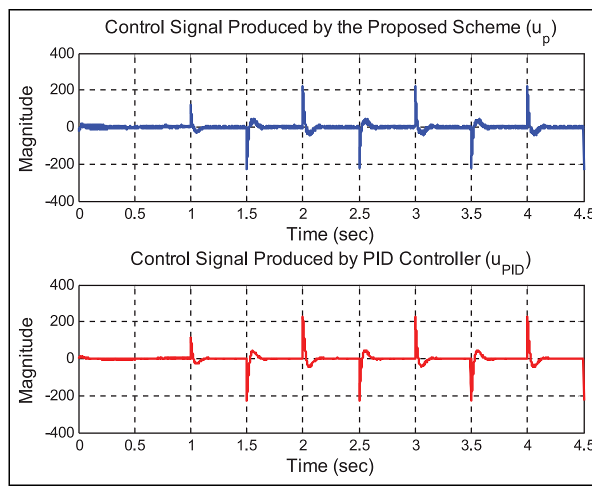

Figure 16. Control signals of the nominal system and the plant with proposed method in presence of the noise. Reference signal is sinusoidal.

of differentiators and clearly makes the closed loop vulnerable to noise and high-frequency disturbances. Our work uses the model-following error and its first derivative in the proposed scheme and the simulations show that the effect of noise is admissible. In Fallahi and Azadi (2009), a PID controller is used to produce the switching variable, and a fuzzy subsystem is used to provide the parameters of the PID controller as well as the coefficients of the sliding term. However, the role share between the fuzzy part and the sole PID controller is not explained. Our paper modifies a nominal PID controller, which has a meaning in some practical applications. A control law can slightly be modified to obtain a good performance with low cost. A similar approach is adopted in Kharabian (2014), where a PID module provides the switching variable and a fuzzy model provides the P, I and D action coefficients. Our approach is structurally different from the cited methods, i.e. using the output of a PID controller as an intermediate variable and differentiating it limits the practical domain of the approach, as in Kharabian (2014). In Dib et al. (2015), a PID control law containing three terms is chosen and the values of the P, I and D action coefficients are provided by a Mamdani fuzzy system. A study of robustness against uncertainties and measurement noise is not given.

We discuss the combination of PID, FLC and SMC in such a way that our work demonstrates:

a) The role of nominal controller with several types of reference signals;
b) The enhancing role of the fuzzy logic part and what happens when it is on and off;
c) The alleviation of complex uncertainties with an analytical proof;
d) Admissible results with measurement noise.

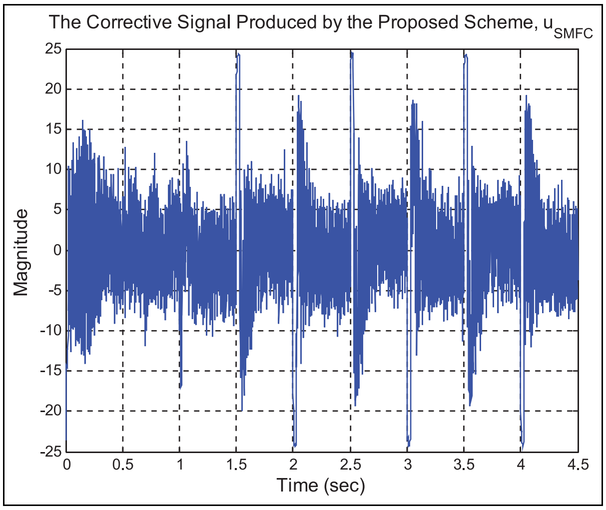

Figure 17. The corrective control signal produced by the proposed scheme. The observations are noisy.

The current work advances the subject area toward the field of minimum modification concept for feedback control systems. This is especially important, as in some cases, a totally new controller is not advised; instead, a minor modification to the already available controller is preferred. This paper contributes a useful solution to the literature.

Concluding remarks

In this paper, by combining the classical PID scheme and the SMFC scheme, a robust control method was presented. The goal of the proposed method is to force the uncertain plant to follow the response of the nominal system. In order to achieve that, first a PID controller was designed for the nominal system by using Ziegler–Nichols method. Then, the designed PID controller was used with the uncertain plant and it was observed that the tracking performance of the uncertain plant was not satisfactory. In the following phase, a fuzzy logic decision mechanism adjusting the SMC magnitude parameter was designed based on the SMC approach. Finally, the control signals of the designed sliding mode fuzzy controller and the PID controller were combined. The prominent features of the proposed scheme have been shown through a set of simulations and it was seen that the loop response could be improved by minor modifications to the original PID control loop. The use of such a mechanism is beneficial in the cases where the nominal controller is a fixed one and the process input is modifiable. Some processes display such properties and the solution offered here is an alternative to obtain a good performance without giving concessions to the quality of the output.

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Conflict of interest

The authors declare that there is no conflict of interest.

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References

Ahmad TA and Zhu Q (2015) *Advances and Applications in Sliding Mode Control Systems*. Berlin: Springer.

Aström KJ and Hägglund T (1995) *PID Controllers: Theory, Design and Tuning*. Research Triangle Park, NC: ISA.

Barrero F (2002) Speed control of induction motors using a novel fuzzy sliding mode structure. *IEEE Transactions on Fuzzy Systems* 10: 375–383.

Bouarroudj N, Boukhetala D and Boudjema F (2015) A hybrid fuzzy fractional order PID sliding-mode controller design using PSO algorithm for interconnected nonlinear systems. *Control Engineering and Applied Informatics* 17(1): 41–51.

Chen Z, Cong BL and Liu XD (2014) A robust attitude control strategy with guaranteed transient performance via modified Lyapunov-based control and integral sliding mode control. *Nonlinear Dynamics* 78(3): 2205–2218.

Dib F, Meziane KB and Boumhidi I (2015) Sliding mode control without reaching phase for multimachine power system combined with fuzzy PID based on PSS. *WSEAS Transactions on Systems and Control*, 206–214.

Edwards C and Spurgeon SK (1998) *Sliding Mode Control: Theory and Applications*. London: Taylor and Francis.

Esfahani HN and Azimirad V (2013) A new fuzzy sliding mode controller with PID sliding surface for underwater manipulators. *International Journal of Mechatronics* 3: 224–249.

Esfahani HN, Azimirad V and Zakeri M (2014) Sliding mode-PID fuzzy controller with a new reaching mode for underwater robotic manipulators. *Latin American Applied Research* 44: 253–258.

Fallahi M and Azadi S (2009) Robust control of DC motor using fuzzy sliding mode control with PID compensator. In: *Proceedings of the International Multi Conference of Engineers and Computer Scientists*, vol. II, 18–20 March, Hong Kong.

Itkis U (1976) *Control Systems of Variable Structure*. New York: Wiley.

Kharabian B (2014) Fuzzy sliding mode-PID control for space manipulator using dynamically equivalent manipulator model. *International Journal of Control and Automation* 7: 143–158.

Lhee CG (2001) Sliding-like fuzzy logic control with self-tuning the dead zone parameters. *IEEE Transactions on Fuzzy Systems* 9: 343–348.

Mamdani EH (1974) Application of fuzzy algorithms for control of simple dynamic plant. *Proceedings of the Institution of Electrical Engineers* 121(12): 1585–1588.

Mohammadi M and Nafar M (2013) Fuzzy sliding-mode based control (FSMC) approach of hybrid micro-grid in power distribution systems. *International Journal of Electrical Power & Energy Systems* 51: 232–242.

Özkop E, Altas IH, Okumus HI, et al. (2015) A fuzzy logic sliding mode controlled electronic differential for a direct wheel drive EV. *International Journal of Electronics* 102(11): 1919–1942.

Papadopoulos K (2014) *PID Controller Tuning Using the Magnitude Optimum Criterion*. Berlin: Springer.

Piltan F, Nabaee A, Ebrahimi MM, et al. (2013) Design robust fuzzy sliding mode control technique for robot manipulator systems with modeling uncertainties. *International Journal of Information Technology and Computer Science (IJITCS)* 5(8): 123–135.

Piltan F, Sulaiman N, Gavahian A, et al. (2011) Design mathematical tunable gain PID-like sliding mode fuzzy controller with minimum rule base. *International Journal of Robotic and Automation* 2(3): 146–156.

Roopaei M, Jahromi M and Zolghadri (2009) Chattering-free fuzzy sliding mode control in MIMO uncertain systems. *Nonlinear Analysis-Theory Methods & Applications* 71(10): 4430–4437.

Saghafinia A, Ping HW and Uddin MN (2014) Fuzzy sliding mode control based on boundary layer theory for chattering-free and robust induction motor drive. *International Journal of Advanced Manufacturing Technology* 71(1–4): 57–68.

Slotine JJE and Li W (1991) *Applied Nonlinear Control*. New York: Publishing Inc.

Tu KY, Lee TT and Wang WJ (2000) Design of a multi-layer fuzzy logic controller for multi-input multi-output systems. *Fuzzy Sets and Systems* 111: 199–214.

Utkin VI (1977) Variable structure systems with sliding modes. *IEEE Transactions Automatic Control* 22: 212–222.

Utkin VI (1992) *Sliding Modes in Control Optimization*. Berlin: Springer.

Vilanova R and Visioli A (eds) (2012) *PID Control in the Third Millennium: Lessons Learned and New Approaches*. Berlin: Springer.

Wai RJ, Lin CM and Hsu CF (2002) Self-organizing fuzzy control for motor-toggle servomechanism via sliding mode technique. *Fuzzy Sets and Systems* 131: 235–249.

Yin S, Yu H, Shahnazi R, et al. (2016) Fuzzy adaptive tracking control of constrained nonlinear switched stochastic pure-feedback systems. *IEEE Transactions on Cybernetics*, doi: 10.1109/TCYB.2016.2521179.

Zadeh LA (1965) Fuzzy sets. *Information and Control* 8(3): 338–353.