# $H_2/H_\infty$-Neural-Based FOPID Controller Applied for Radar-Guided Missile

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***Abstract*—In this paper, a neural tuning technique is proposed and applied to a fractional-order proportional-integral-derivative (FOPID) controller. The proposed controller is applied to a radar-guided missile which is used for tracking high-speed moving targets in defense systems. The proposed neural tuning along with $H_2/H_\infty$ optimization method is intended to improve the tracking performance of the proportional navigation (PN) system and the stability of the missile trajectory during flight time. Due to the coupled and nonlinear dynamics of the considered missile, we propose a new control structure that integrates the FOPID controller into the PN system of the missile which results in better stability properties for the missile. In order to tune the FOPID, we propose a neural tuning technique that starts with a genetic algorithm-based optimizer and continues with a neural network-based scheme. This speeds up finding a near-optimal solution and refining it effectively. Then, $H_2/H_\infty$ optimization process is applied within the neural tuning technique to achieve better stability and tracking performance for the missile during the whole flight time. The proposed controller is compared with the standard PID controller tuned by the conventional Ziegler–Nichols (ZN) tuning method as well as particle swarm optimization (PSO) method, and the simulation results proved the superiority of the proposed tuning method over ZN- and PSO-based tuning approaches.**

***Index Terms*—Fractional-order proportional integral-derivative (FOPID), $H_2/H_\infty$, miss distance, neural tuning, proportional navigation, radar-guided missile.**

## I. Introduction

FLIGHT stability and hitting accuracy of missiles are of the most interesting control problems to which scientists have been trying to propose novel solutions for years. There are many performance tests that have been improved by scientists in the field of missile defense systems, and miss distance is one of these important performance tests that indicates the accuracy of missiles. Miss distance could be defined as "the minimum distance between a guided flying object and its intended target site during their intersection" [1]. The nonlinear dynamics of the missile entails a sophisticated controller such as a fractional-order proportional-integral-derivative (FOPID), which performs significantly better than its integer-order counterpart under certain circumstances. A widely accepted approach is to linearize the system under control at a number of operating points and then to perform a scheduling mechanism in between the computed gains. This can be extended to dedicated PID controllers for operating points, and it has been reported in the literature that using a fractional-order controller could be a remedy to obtain better stability properties and better tracking performance.

The dynamics of a missile is highly nonlinear, and the variables are inextricably intertwined. Demanding requirements on the closed-loop system motivates the use of a fractional-order solution approach for stability and hitting accuracy. However, achieving an effective tuning scheme for the fractional-order system is a challenging problem especially when dealing with uncertainties or unpredictable environmental changes. Therefore, we propose a novel tuning method that is based on constructing a lookup table, which contains a set of FOPID parameters as well as the miss distance value. These have been obtained by implementing the FOPID parameters, and then, a deep learning technique is employed to learn the relation between the FOPID parameters and the miss distance value. After the learning process, the neural network predicts the best parameters of the FOPID that could achieve the best stability and minimum miss distance. In conjunction with this, the stability of this proposed method is increased by applying $H_2/H_\infty$ optimization process along with the miss distance. Proportional navigation (PN) system is considered as the most preferred system for guiding the missile toward a specific target [2]. Therefore, many researchers investigated the possibility of integrating new types of controllers into the PN system. In [3], an adaptive sliding-mode controller exploiting neural networks is presented, and the approach is applied to a class of uncertain strict-feedback system. A radial basis function neural network is used in order to decrease the chattering resulted from sliding mode controller (SMC). The simulation results demonstrated the effectiveness and robustness of the proposed control technique in decreasing the chattering caused by the sliding-mode control scheme. In [4], a fuzzy tracking control is applied to a near-space hypersonic vehicle which is subjected to stochastic actuator failures as well as aperiodic measurement information. In this paper, a reliable fuzzy tracking control strategy is introduced and the validity of the proposed system is confirmed via simulation results. In [5], a self-tuning PID controller based on fuzzy wavelet neural network is proposed

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and applied to a nonlinear system. The proposed control scheme effectively handled the limitations of the PID controller when dealing with unknown systems or unpredictable changes in the environment of the system. Prominent features of the control system in [5] have been shown by a series of simulations. In [6], the authors addressed the problem of finite horizon fault estimation for two-dimensional discrete time-varying systems that are specified with measurement noise and bounded unknown input. The effectiveness of the proposed technique is proved by using a thermal process example. In [7], a sliding-mode control scheme based on the time-specified nonsingular terminal is proposed. The proposed controller is applied on robotic airships for trajectory tracking problem. The experimental results showed the effectiveness of the proposed controller that avoids the singularity problem of the sliding-mode controller as well as it specifies the convergence time. In [8], a new guidance law is proposed for intercepting aggressively maneuvering targets. The autopilot dynamics have been modeled as a first-order transfer function, and the simulation results showed good performance for the proposed guidance law for highly maneuvering targets. In [9], an improved control scheme is studied in order to realize the nonlinear decoupling and tracking of the angle-and-roll motion. The enhanced method is based on trajectory linearization control scheme and an improved control law which is implemented by the principle of time-scale separation. The simulation studies proved the effectiveness of the adaptive decoupling control in ensuring the tracking performance, accuracy, and robustness of the missile attitude under uncertainty, external disturbances, and observation noise. In [10], a new guidance law is proposed for estimating the acceleration demands of the PN guidance system. The simulation results demonstrated the effectiveness of the investigated guidance law. In [11], a neuro-genetic hybrid approach is adopted for tuning a FOPID controller. The control system in [11] is compared with the conventional PID control system in terms of miss distance, $H_2/H_\infty$ measure, missile trajectory tracking performance, and angle of attack values. The suggested approach displayed better performance with the FOPID controller when compared to the conventional PID scheme. The outcomes reported in [12] integrate the PID controller into missile navigational guidance system. The purpose of integrating the PID is to improve the miss distance accuracy of the PN system. However, the PN guidance system is a nonlinear system, and the capabilities of the PID controller in such a nonlinear control system may not be satisfactory. Therefore, we introduce the use of FOPID which has more degrees of freedom than its integer-order counterpart.

The rest of this paper is organized as follows. Section II contains a review of the PN guidance system. Section III describes the missile dynamics and equations of motions. Section IV introduces the mathematical model of the proposed FOPID controller. Section V discusses the conventional tuning method applied to the PID controller as well as the proposed tuning method for the FOPID. Section VI contains an analysis of the simulation results and compares the proposed FOPID against the integer-order PID. Section VII concludes this paper.

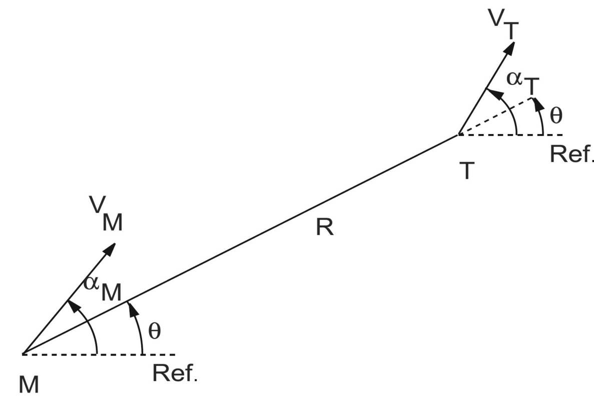


Fig. 1. Missile–target engagement geometry.

## II. Proportional Navigation System

The main concept of the PN system is about finding a value of normal acceleration to correct the deviation of the missile from the line of sight (LOS), by that, the PN guidance system will work on keeping the rotation rate of the missile equals the rotation rate of the LOS. The kinematic model of a missile deviated from the LOS is shown in Fig. 1.

A normal acceleration should be applied in order to correct the missile direction and align the missile with the LOS. The value of normal acceleration is related to the distance between the missile and the LOS. The normal acceleration value $a_M$ that is needed to be applied on the missile to keep it aligned with the LOS could be found using the following equations:

$$V_r = \dot{R} = V_T \cos(\alpha_T - \theta) - V_M \cos(\alpha_M - \theta) \quad (1)$$

$$V_\theta = R\dot{\theta} = V_T \sin(\alpha_T - \theta) - V_M \sin(\alpha_M - \theta) \quad (2)$$

$$\dot{\alpha}_M = \frac{a_M}{V_M} \quad (3)$$

$$a_M = N V_M \dot{\theta} \quad (4)$$

where $R$ is the distance between the missile and the target, $\alpha_T$ is the angle between the moving direction of the target and the reference axis, $\alpha_M$ is the angle between the moving direction of the missile and the reference axis, $\theta$ is the angle between the target position with respect to the missile and the reference axis (angle between the reference axis and the LOS), $N$ is the navigation constant, and $a_M$ is the lateral acceleration of the missile.

## III. Missile Dynamics and Equations of Motion

The dynamics of the missile could be obtained by finding the forces applied on each axis as well as the pitch moment. The following equations describe the dynamics of the missile:

$$F_x = C_x \left(0.5\rho V^2 S_{\text{ref}}\right) \quad (5)$$

$$F_z = C_z \left(0.5\rho V^2 S_{\text{ref}}\right) \quad (6)$$

$$M = (C_m + q)\left(0.5\rho V^2 S_{\text{ref}} D_{\text{ref}}\right) \quad (7)$$

where $F_x$ is the force projected on the $x$-axis of the missile, $F_z$ is the force projected on the $z$-axis of the missile, $M$ is the pitch moment exerted by the missile, $S_{\text{ref}}$ is the value of the missile reference cross-sectional area, $D_{\text{ref}}$ is the diameter of the reference circular body of the missile, $V$ is the speed of the missile, $C_x$ and $C_z$ are coefficients that have values relative to the speed and angle of attack of the missile, and these values are stored in lookup tables. The forces and pitch moment values are then used to generate the acceleration of the missile with respect to each axis as well as the rate of change in the angle of attack. With these definitions, the trajectory of the missile could be obtained as given in (8)–(11)

$$A_x = \frac{F_x}{m} - qv_x - g\sin(\theta) \tag{8}$$

$$A_z = \frac{F_z}{m} + qv_z + g\cos(\theta) \tag{9}$$

$$\dot{q} = \frac{M}{I} \tag{10}$$

$$\dot{\theta} = q \tag{11}$$

where $A_x$ is the acceleration of the missile projected on the $x$-axis, $A_z$ is the acceleration of the missile projected on the $z$-axis, $\theta$ is the missile pitch angle, $q$ is the rotation rate of the missile body, $m$ is the mass of the missile, $v_x$ is the speed of the missile projected on the $x$-axis of the missile reference frame axes, $v_z$ is the speed of the missile projected on the $z$-axis of the missile reference frame axes, $g$ is the gravity force, and $I$ is the inertia of the missile.

## IV. FOPID Scheme

Fractional calculus is a branch of mathematics that generalizes the definition of integration and derivation with integer orders to fractional orders. The history of fractional calculus dates back to 1695 when L'Hôpital wrote a letter to Leibniz asking about the meaning of a derivative with an order of 1/2. However, and due to computational issues, the real-time control implementations of fractional-order systems have not been possible until the last few decades. The integer-order PID controller is the most preferred control scheme in the industry due to its simplicity and ease of implementation. The FOPID controller which could be referred to as $PI^{\lambda}D^{\mu}$ is considered as a generalization of the integer-order PID controller. The transfer function of the FOPID is given in (12)

$$C(s) = \frac{D(s)}{U(s)} = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu}. \tag{12}$$

where $\lambda$ is the integration order and $\mu$ is the differentiation order, and both of them are real positive and noninteger numbers. In the literature, the differintegration operator ${}_{\alpha}D_t{}^q$ is used to define the fractional-order integration or differentiation as given in (13)

$$\alpha D_t^q \begin{cases} \frac{d^q}{dt^q}, & q > 0 \\ 1, & q = 0 \\ \int_{\alpha}^{t} (d\tau)^{-q}, & q < 0 \end{cases}. \tag{13}$$

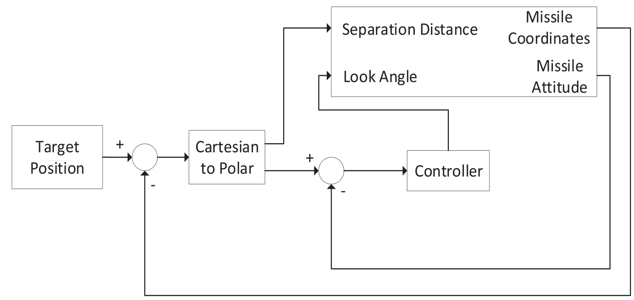


Fig. 2. Control system applied to PN guidance system.

In the past, several works were reported on approximating the differintegration operator; Riemann–Liouville and Caputo's definitions are among the most adopted formulas as given in (14) and (15), respectively

$$D^{\alpha}y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)}\left(\frac{d}{dt}\right)^{r}\int_0^t \frac{y(\xi)}{(t-\xi)^{\alpha+1-r}}d\xi \tag{14}$$

$$D^{\alpha}y = y^{(\alpha)} := \frac{1}{\Gamma(r-\alpha)}\int_0^t \frac{y^{(r)}(\xi)}{(t-\xi)^{\alpha+1-r}}d\xi. \tag{15}$$

The Riemann–Liouville definition lacks the ability to deal with Laplace transform because the noninteger-order derivative at $(t = 0)$ needs to be known in advance, while the approximation technique made by Caputo successfully overcame this problem [13]. In the literature, several approaches were used to approximate the fractional-order operator. Oustaloup's approximation is one of them, which is known for its accuracy. Therefore, it is adopted in the implementation of our FOPID controller discussed in the next section. FOPID controller is superior to the integer-order one in many aspects including stability and robustness [14]. FOPID has better ability in dealing with uncertainties and variations in the gain. It also outperforms the integer-order PID in dealing with load disturbances. FOPID also rejects the noises affecting the system better than the conventional PID [15]. The performance of the FOPID is comparably better than the integer-order one in control systems that involve time delays. Moreover, when using a PID controller for controlling a nonlinear system, the system is usually linearized at a set of operating points, and for each of these points, a specific PID is optimized and tuned, while it is sometimes sufficient to use a single FOPID for that purpose [16].

## V. Controller Design

In this paper, a FOPID control system is proposed and integrated into the PN guidance system; the proposed control system has been tested and compared with the standard PID as shown in Fig. 2.

The tuning process of the standard PID controller is done using the Ziegler–Nichols tuning method as well as particle swarm optimization (PSO) method. The tuning process for ZN started by linearizing the system at a suitable operating point and obtaining a transfer function approximation for the system at that point, and then the controller parameters are obtained using

TABLE I
PARAMETERS PRESCRIBED BY ZIEGLER-NICHOLS TUNING METHOD

| Parameter | PID Controller |
|---|---|
| $K_p$ | 0.1075 |
| $K_i$ | 1.6796876 |
| $K_d$ | 0.0016555 |

TABLE II
PARAMETERS PRESCRIBED BY PSO METHOD

| Parameter | PID Controller |
|---|---|
| $K_p$ | 0.8631 |
| $K_i$ | 0.0010 |
| $K_d$ | 0.0010 |

the Ziegler–Nichols tuning method. The resulting parameters for the PID controller are shown in Tables I and II.

Numerical realization of the proposed FOPID system is designed and implemented using FOMCON toolbox [17], where Oustaloup's approximation method is used for approximating the fractional-order operator $(s^\gamma)$ as described by the following equations:

$$s^\gamma \approx \omega_h^\gamma \prod_{k=-N}^{N} \frac{s+\omega'_k}{s+\omega_k} \tag{16}$$

where

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}} \tag{17}$$

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}} \tag{18}$$

where $0 < \gamma < 1$, and $N$ is the approximation order and the operating frequency range lies between $\omega_b$ and $\omega_h$. In order to observe a good closed-loop control performance, high $N$ and wide $\frac{\omega_h}{\omega_b}$ are desired. Yet, this increases the computational complexity, and therefore an admissible approximation should be sought. The chosen frequency range should cover the operating frequencies of the plant and $N$ should be high enough to observe a flat magnitude response in decibels. In this paper, the value of the approximation order $N$ is 5 and the frequency range lies between [0.001 and 1000] Hz. Having these in mind, the major FOPID controller parameters that need to be tuned are $K_i$, $K_p$, $K_d$, λ, and $\mu$. The tuning process for the FOPID controller has been done using genetic algorithm, which is fast at the initial stage, because of that, the genetic algorithm is kept active until a critical miss distance value is reached. In this paper, we considered a miss distance value of 0.1 m as the critical value. The simulation successfully achieved a miss distance value less than 0.1 m after the seventh generation. Around the optimum value, genetic algorithm needs a long time to converge, and therefore, the algorithm switches to neural network-based optimization for fine-tuning. The controller parameters that have been obtained using genetic algorithm are shown in Table III.

TABLE III
PARAMETERS OBTAINED BY GENETIC ALGORITHM

| Parameter | FOPID Controller |
|---|---|
| $K_p$ | 0.7094 |
| $K_i$ | 0.2046 |
| $K_d$ | 0.2048 |
| $\lambda$ | 0.4165 |
| $\mu$ | 0.1531 |

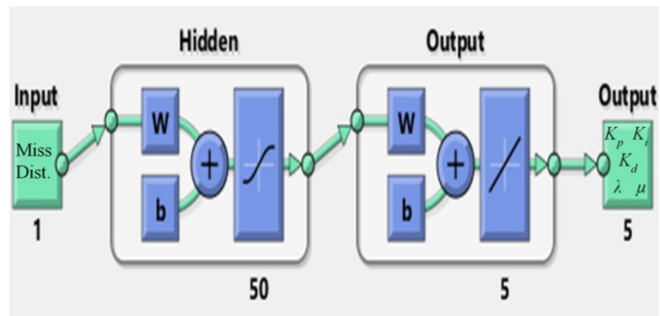


Fig. 3. The proposed neural tuning technique for the FOPID parameters.

Using these parameters in the FOPID controller, the resulted miss distance was observed as 0.0168 m, which is better than using the standard PID that yielded a miss distance value of 25.48 m. In the neural network-based optimization stage, the input and the outputs of the neural network are defined as follows:

$$\text{Input} = \begin{bmatrix} \text{Miss Distance}_1 \\ \vdots \\ 0.0168 \\ \vdots \\ \text{Miss Distance}_N \end{bmatrix} \tag{19}$$

$$\text{Output} = \begin{bmatrix} K_{p1} & K_{i1} & K_{d1} & \lambda_1 & \mu_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.7094 & 0.2046 & 0.2048 & 0.4165 & 0.1531 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_{pN} & K_{iN} & K_{dN} & \lambda_N & \mu_N \end{bmatrix}. \tag{20}$$

The input for the neural network is the miss distance value while the outputs are the parameters of the FOPID controller, i.e., $K_p$, $K_i$, $K_d$, λ, and $\mu$ as shown in Fig. 3. These values are the training pairs obtained during the genetic optimization process. A feed-forward neural network type was used with two layers, and 50 neurons in the hidden layer.

The training process was started using the Levenberg–Marquardt algorithm, while the initial biases and weights were obtained using the Nguyen–Widrow algorithm. The training process and results are shown in Table IV.

The network was then tested by entering a small value of miss distance near to zero (e.g., 0.0001 m); then the network

TABLE IV
TRAINING PROCESS AND RESULTS

| *Process* | *Samples* | *Mean Squared Error* | *Regression* |
|---|---|---|---|
| *Training* | *2187* | $5.60698*10^{-7}$ | $9.99995*10^{-1}$ |
| *Validation* | *469* | $5.81941*10^{-7}$ | $9.99995*10^{-1}$ |
| *Testing* | *469* | $5.93842*10^{-7}$ | $9.99995*10^{-1}$ |

TABLE V
PARAMETERS OBTAINED BY THE NEURAL NETWORK TUNING METHOD

| Parameter | FOPID Controller |
|---|---|
| $K_p$ | 0.9368 |
| $K_i$ | 0.0112 |
| $K_d$ | 0.0419 |
| $\lambda$ | 0.1738 |
| $\mu$ | 0.0588 |

predicted the optimal FOPID parameters that produce the desired miss distance value. It should be noted that the obtained neural network is not unique, and a well-trained neural network structure is able to generate the needed FOPID parameters. The structure shown in Fig. 3 states that the five parameters of FOPID are nonlinear functions of the miss distance.

Using the proposed neural network-based approach, the resulted control parameters for a 0.0001 miss distance value are listed in Table V.

Implementing the FOPID controller with the parameter listed in Table IV, the measured miss distance is 0.0047 m, which is significantly more accurate than the results obtained using genetic algorithm technique as well as the ZN conventional tuning method. After the neural network-based tuning, a final refinement can be carried out using $H_2/H_\infty$-based tuning method. The $H_2$ norm for a scalar-valued signal $u(t)$, or $u_2$, is defined as in (21)

$$\|u_2\| = \left(\int_0^\infty u(t)^2 dt\right)^{1/2}. \tag{21}$$

The physical interpretation of the $H_2$ norm is that if $u(t)$ represents a signal, then $u_2^2$ is proportional to the total energy associated with that signal. Infinity norm for a scalar-valued signal $u(t)$, or $u_\infty$, could be obtained by finding the maximum of the absolute value of the signal $u(t)$ as in (22)

$$\|u_\infty\| = \max_t |u(t)|. \tag{22}$$

Infinity norm is the maximal value in the output of the system and it is used in order to optimize the system with guaranteed performance for the whole range of frequencies [18]. In the $H_2/H_\infty$ optimization process, the values of $H_2$ and $H_\infty$ norms are added to the training matrix together with miss distance values as shown in (23) and (24), where $P$ is the number of training samples. In this approach, the neural network structure obtains the FOPID parameters as functions of miss distance, 2-norm value, and $\infty$-norm value

$$NN_{\text{Input}} = \begin{bmatrix} \text{Miss Distance}_1 & H_{21} & H_{\infty 1} \\ \vdots & \vdots & \vdots \\ 0.0168 & 6.8167 & 3.2757 \\ \vdots & \vdots & \vdots \\ \text{Miss Distance}_P & H_{2P} & H_{\infty P} \end{bmatrix} \tag{23}$$

$$NN_{\text{Output}} = \begin{bmatrix} K_{p1} & K_{i1} & K_{d1} & \lambda_1 & \mu_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0.7094 & 0.2046 & 0.2048 & 0.4165 & 0.1531 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K_{pP} & K_{iP} & K_{dP} & \lambda_P & \mu_P \end{bmatrix}. \tag{24}$$

TABLE VI
PARAMETERS GENERATED BY THE $H_2/H_\infty$ OPTIMIZATION

| Parameter | FOPID Controller |
|---|---|
| $K_p$ | 0.9434 |
| $K_i$ | 0.0133 |
| $K_d$ | 0.0421 |
| $\lambda$ | 0.1122 |
| $\mu$ | 0.0278 |

After the learning process is completed, the FOPID parameters are obtained by entering small values near to zero (e.g., 0.0001) for $H_2$ and $H_\infty$ as well as miss distance, so that, the neural network will output a set of FOPID parameters that could achieve the prescribed small values of $H_2$, $H_\infty$, and miss distance. The resulting FOPID parameters proved to achieve further improvement for the stability of the missile in terms of 2- and $\infty$-norms as seen in the results section. The resulted FOPID parameters obtained from $H_2/H_\infty$ optimization are listed in Table VI.

The tuning process of the proposed FOPID controller could be summarized in Algorithm 1 as shown below.

## VI. PERFORMANCE ANALYSIS

### A. Missile Trajectory

The trajectories of the missile and the target using ZN and PSO tuning methods of the PID controller, as well as GA and neural-$H_2/H_\infty$ tuning methods of the FOPID controller, are shown in Fig. 4. It is clear from the figure that the system with the integer-order PID controller, which was tuned using ZN and PSO methods, performs poorly compared to all other alternatives. Oscillations were observed during the flight time and the missile missed the target. GA tuning method applied to FOPID controller proved to be much better, where the course of the missile is much more stable and displays no oscillations. The missile also successfully hit the target with high accuracy. Using the proposed neural-$H_2/H_\infty$-tuned FOPID controller, the missile trajectory to the target is shorter than the other methods. The followed path is smooth and the accuracy is high.

**Algorithm 1:** The Neural Tuning Algorithm of the FOPID Controller.

**Input:** Random values for the FOPID parameters.
**Output:** Optimal values for the FOPID parameters.
**Start:**
Step 1: Set the initial values for the parameters of the FOPID controller ($K_p$, $K_i$, $K_d$, $\mu$, and $\lambda$).
Step 2: Perform a closed-loop test to evaluate the miss distance value based on initial controller parameters.
Step 3: Minimize the miss distance value using GA.
Step 4: Construct a lookup table that contains a different set of FOPID parameters around the generated values from GA.
Step 5: Calculate the miss distance, 2-norm, and $\infty$-norm for each set of FOPID parameters and add them to the lookup table.
Step 6: Train a neural network by considering the values of miss distance, 2-norm, and $\infty$-norm as inputs to the neural network, and the FOPID parameters as an output.
Step 7: Obtain the final values of the FOPID parameters (output of the neural network) by entering small values to the inputs of the neural networks (miss distance, 2-norm, and $\infty$-norm).
**End.**

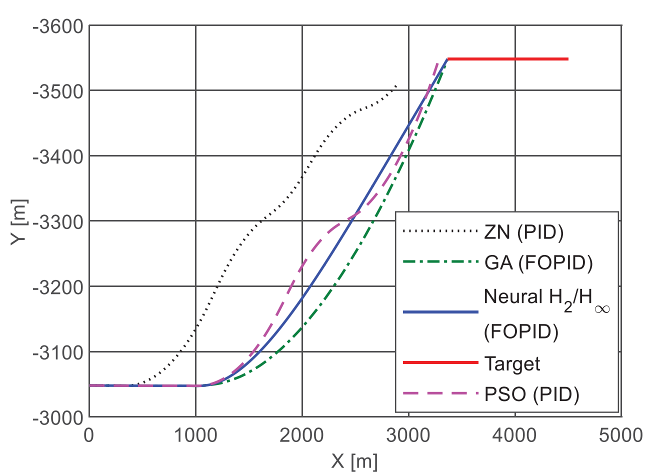


Fig. 4. Missile and target trajectories using ZN, GA, and neural $H_2/H_\infty$ controller for the missile.

## B. Angle of Attack

The behavior of the angle of attack for the missile using ZN tuning method is shown in Fig. 5. As shown in the figure, the angle of attack is oscillating approximately between 20 and $-20$ degrees during the flight time, and that indicates instability which is undesired. A natural consequence of the shown oscillation is the increase in the drag forces applied on the missile and the speed of the missile will therefore be decreased.

Fig. 6 shows the performance of the angle of attack for the missile using PSO tuning method. The system has fewer oscillations compared to ZN, but it is still unstable.

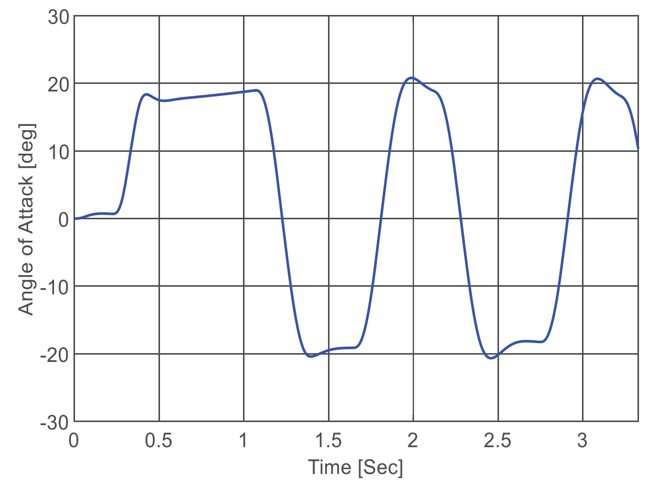


Fig. 5. Angle of attack for the missile using ZN tuning for PID controller.

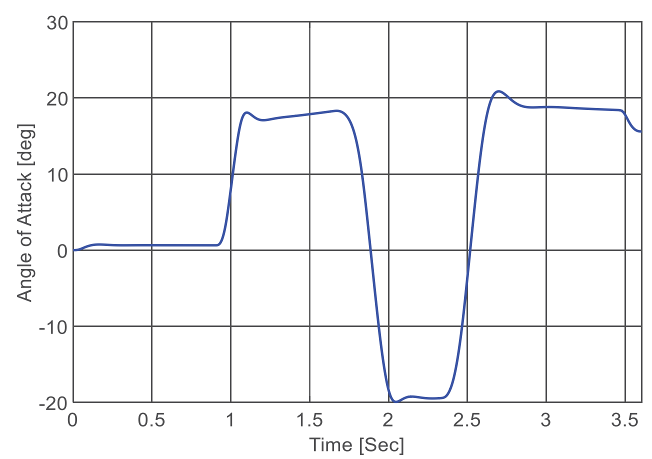


Fig. 6. Angle of attack behavior for the missile using PSO tuning for the PID controller.

Fig. 7 shows the angle of attack for the missile using the proposed neural-$H_2/H_\infty$ tuning method. The results obtained by this tuning method were observed to be much more stable than PSO and ZN. The angle reached its highest value when the radar system has found the target, then it has decreased smoothly until the target has been intercepted by the missile.

## C. Miss Distance

The proposed control method has been simulated and compared against the conventional method in terms of miss distance. The miss distance value for the ZN-based control system was 25.48 m, while using the PSO tuning method on the PID controller, the miss distance value was 11.60 m. The proposed tuning procedure started by applying the GA on the FOPID controller, which is fast for obtaining near-optimal values at initial stages. At the end of this stage, the resulted value of miss distance was 0.0168 m. The second stage employs a neural network-based refinement. The training process is fast and more accurate than the genetic algorithm near the optimal values. The miss distance after this stage was observed as 0.0047 m.

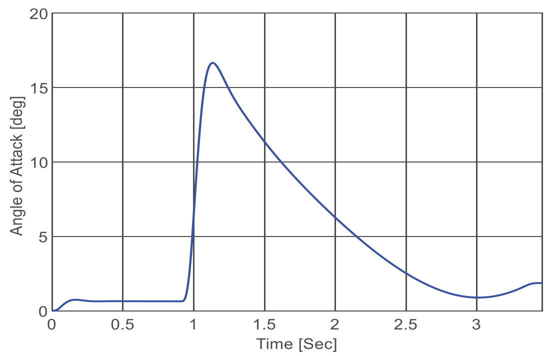


Fig. 7. Angle of attack for the missile using $H_2/H_\infty$ neural tuning of the FOPID controller.

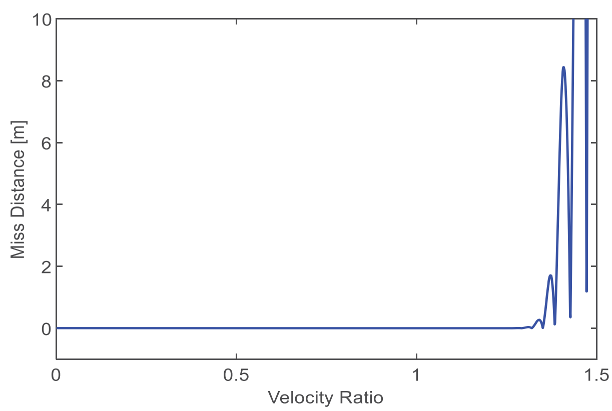


Fig. 8. Miss distance vs. velocity ratio for multitrajectories.

After that, the values obtained from the neural tuning method were optimized more using $H_2/H_\infty$-based tuning process. The final obtained miss distance using $H_2/H_\infty$ optimization method was 0.0262 m. Although the miss distance was decreased only a little bit by using $H_2/H_\infty$-based tuning process, it is very accurate. Also, the overall control performance is better using $H_2/H_\infty$ tuning process as seen in the next section. These values proved the accuracy of the proposed tuning method over the standard PID in terms of miss distance. Fig. 8 shows the miss distance values for multitarget trajectories. The miss distance value was near to zero where the ratio of the target velocity to the missile velocity was between 0 and 1.35. By that, we can confirm that the miss distance for the proposed controlling system was kept near to zero for the whole velocity ratios needed by the missile (velocity of the target/velocity of the missile). In real applications, the velocity of the missile is usually much faster than the velocity of the target.

### D. $H_2$ and $H_\infty$ Norms

The proposed FOPID controller has been compared against the standard PID controller tuned by the conventional ZN and PSO methods in terms of 2-norm and $\infty$-norm of the error signals. The 2-norm of the standard PID controller tuned by ZN has been computed and it was 13.4522, while using PSO on the same controller, the 2-norm value decreased to 11.2192. Using GA on the proposed FOPID, the 2-norm value was observed as 6.8167.

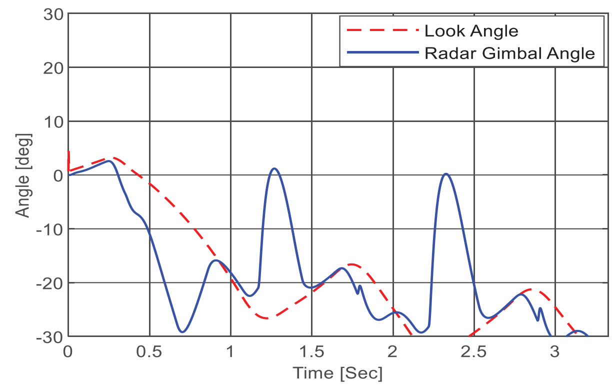


Fig. 9. Look angle vs. radar gimbal angle for ZN tuning of PID controller.

After applying the neural network-based tuning technique, the 2-norm value decreased to 5.2491. Finally, after applying $H_2/H_\infty$ optimization process, the value of the 2-norm was 4.1859 which accounts for about 69% increment in the performance over ZN tuning method, and about 20% over the neural tuning method. The $H_\infty$ value of the PID controller after applying ZN tuning method was observed as 0.4893, while it was 0.4478 after applying PSO tuning method on the same controller. By using the proposed FOPID controller tuned by GA, the infinity norm value was observed as 3.2757. This indicates that using genetic algorithm, the performance of the $\infty$-norm decreased. After applying neural tuning process, the $\infty$-norm was 3.0959. After applying $H_2/H_\infty$ optimization process, the resulting infinity norm was observed as 0.1654 which accounts for 66% increment in the performance over the ZN method, and about 95% increment in the performance over neural technique. By that, the ability of the $H_2/H_\infty$ optimization process in stabilizing the missile during flight time is proved, and the problem of increasing the values of the 2-norm and $\infty$-norm caused by genetic and neural tuning techniques is solved by $H_2/H_\infty$ optimization process. These values prove the performance and stability of the proposed neural-$H_2/H_\infty$ FOPID controller which is superior to the standard PID tuned by the Ziegler–Nichols method.

### E. Look Angle vs. Radar Gimbal Angle

In missile guidance system, the used closed-loop control algorithm has a critical impact on the performance of the radar system. The radar system should be able to track the target and align the radar gimbal angle with the look angle. Using the ZN tuning method, the missile was oscillating during the flight which makes the look angle to be oscillating too, as shown in Fig. 9. Also, the performance of the radar gimbals in tracking the look angle is poor as shown in the figure.

Fig. 10 shows the look and gimbal angles for the missile using PSO tuning method. As shown in the figure, the look angle is also oscillating due to the oscillation of the missile's body during flight, but the radar performance in tracking this oscillation and aligning the gimbal angle with the look angle is good.

Fig. 11 shows the look and gimbal angles for the missile using the proposed neural-$H_2/H_\infty$ tuning. It is clear from the figure that the missile has no oscillation as the look angle is almost

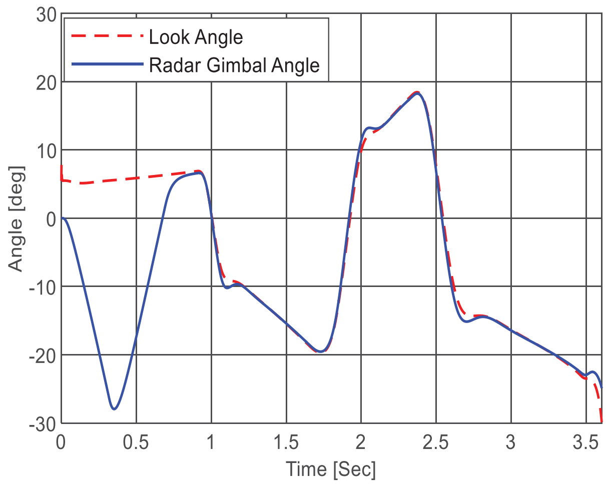


Fig. 10. Look angle vs radar gimbal angle for PSO tuning of the PID controller.

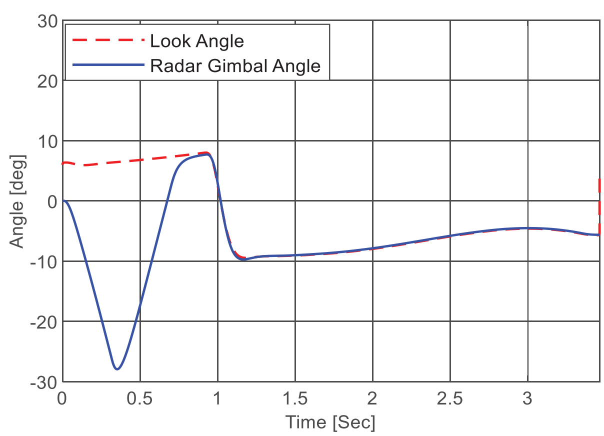


Fig. 11. Look angle vs radar gimbal angle for neural-$H_2/H_\infty$ tuning of FOPID controller.

smooth for most of the time after finding the target (nearly 1 s) and the radar gimbal angle is tracking the look angle accurately. For the considered problem, this proves the superiority of the proposed neural-$H_2/H_\infty$ tuning method over ZN tuning method.

## F. Noise and Disturbances

The proposed control system is re-tested under the effect of noise. The performance results for the proposed FOPID under these conditions are demonstrated in Figs. 12 and 13. As seen in Fig. 12, the noise and disturbances added to the system have affected the acceleration demands; however, the adverse effects of noise have been suppressed appropriately, and the real acceleration exerted by the system displayed a smooth behavior.

The behavior of the angle of attack under noise and disturbances is shown in Fig. 13. According to the results, we can conclude that the controller is able to control the system efficiently under noise and disturbances, and the performance of the proposed system under these conditions is still much better than the standard controller without applying any noise or disturbances.

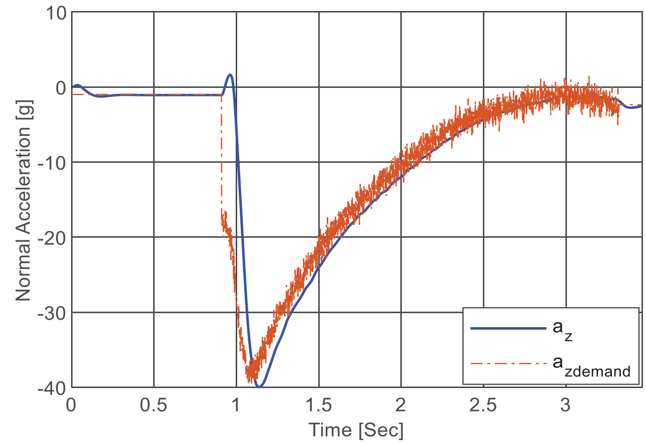


Fig. 12. Effect of noise on normal acceleration and demanded normal acceleration for the proposed FOPID.

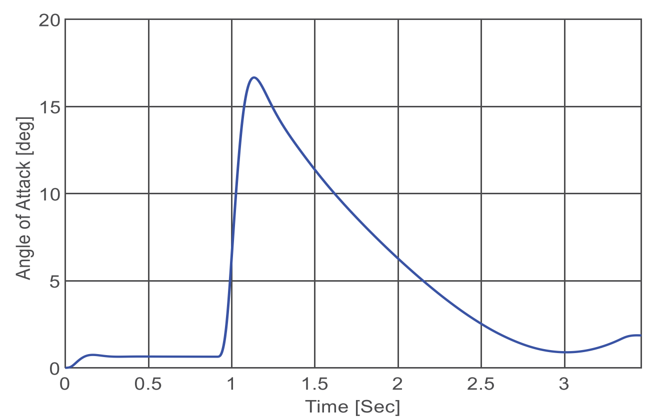


Fig. 13. Effect of noise on the angle of attack for the proposed FOPID.

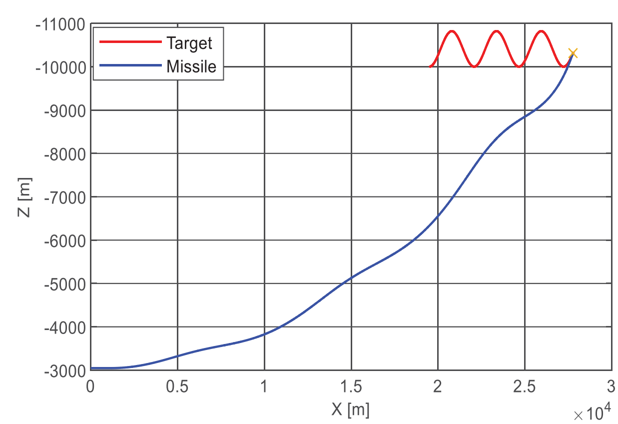


Fig. 14. Trajectory of the missile against a maneuvering target using the proposed FOPID.

## G. Maneuvering Target

The missile is also tested against a maneuvering target. Fig. 14 shows the trajectory of the missile using the proposed FOPID against a maneuvering target with a sinusoidal movement course.

## VII. Conclusion

In this paper, a neural-$H_2/H_\infty$-based FOPID controller was proposed and applied for radar-guided missile. The goal was to guide the missile toward the target with high tracking performance and good hitting accuracy. The proposed controller was tuned in three stages. The first stage utilized a genetic algorithm technique for tuning the FOPID controller, which was supposed to have fast convergence speed at initial stages, and slow convergence speed near optimal values. The second stage employed a novel neural network tuning technique to the FOPID control system, which had better accuracy than the previous genetic algorithm and faster convergence speed near the optimal values. Finally, an $H_2/H_\infty$ optimization technique was introduced for better tracking performance during flight time. The proposed FOPID controller was tested and compared with the standard PID controller that was tuned by the Ziegler–Nichols tuning method as well as a PID controller that was tuned by PSO method in terms of missile trajectory stability, miss distance, angle of attack, radar tracking of the true look angle, 2-norm, and $\infty$-norm. The simulation results showed that the FOPID controller with the proposed novel tuning method leads to better accuracy in hitting the target with low miss distance. It also generated a smooth trajectory displaying no oscillations, unlike the conventional PID controller that has a visible sequence of oscillations during the flight. The performance of the gimbal angle of the radar in tracking the true look angle was much better using the proposed FOPID controller. The proposed $H_2/H_\infty$ optimization technique increased the performance of the infinity norm to a much better value than the ZN method, in which it had about 95% increment in the performance over the neural tuning method alone, and about 66% increment in the performance over ZN tuning method. Also, the proposed $H_2/H_\infty$ optimization technique accounted for 69% increment in the performance over ZN tuning method, and about 20% over the neural tuning method in terms of the 2-norm. The change in the angle of attack was much more stable for the FOPID. Also, the 2- and $\infty$-norms ($H_2$ and $H_\infty$) for the FOPID controller were lower than the conventional PID controller, which indicates the stability of the FOPID controller in controlling the missile during the whole flight time. Finally, the proposed controller was tested under noise, disturbances, and against a maneuvering target, and it showed smooth behavior under these conditions. These results could influence future work by applying the Kalman filter which is supposed to improve the ability to reject random noise that the control system might face. Also, extending the proposed optimization technique to fault estimation problem is supposed to increase the stability of the control system.

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