Adaptive Fault-Tolerant Control of a Probe-and-Drogue Refueling Hose Under Varying Length and Constrained Output

Zhijie Liu, *Member, IEEE*, Jinyuan Liang, Zhijia Zhao, *Member, IEEE*, Mehmet Önder Efe, *Senior Member, IEEE*, and Keum-Shik Hong, *Fellow, IEEE*

***Abstract*— This brief deals with an adaptive barrier-based fault-tolerant control of a probe-and-drogue refueling hose (PDRH) under varying length, actuator fault, and constrained output. First, we consider the actuator working normally, and a model-based control law with barrier function is developed to stabilize the elastic vibration and ensure the endpoint vibration without violating the restriction of the flexible hose. Second, an adaptive fault-tolerant control scheme is proposed to cope with partial effectiveness loss of the actuator. Subsequently, the direct Lyapunov method is adopted to derive the uniformly bounded stability in the closed-loop system. Finally, simulation results demonstrate the validity of the obtained schemes.**

***Index Terms*— Adaptive control, fault-tolerant control, output constraint, probe-and-drogue refueling hose (PDRH), vibration control.**

Nomenclature

ρ	Linear density of the refueling hose.
C	Specified boundary constraint of the refueling hose.
$d(t)$	Boundary disturbance at the endpoint position of the refueling hose.
$d_d(z,t)$	Distributed disturbance along the refueling hose, where $0 < z < E(t)$.
$E(t)$	Length of the refueling hose varied along time t.
$f_h(z,t)$	Surface friction of the refueling hose in the tangential direction, where $0 < z < E(t)$.
$f_{\text{drog}}(t)$	Resistance of the drogue.
$f_n(z,t)$	Pressure exerted on the refueling hose along the normal direction, where $0 < z < E(t)$.
g	Gravitational acceleration.
L	Maximum length of the refueling hose.
m	Weight of the drogue.
$T(E(t),t)$	Endpoint position tension of the refueling hose with respect to time t, where $0 < E(t) \leq L$.
$T(z,t)$	Tension of the refueling hose at position z for time t, where $0 < z < E(t)$.
$v(t)$	Advancing velocity of the tanker.
$w(E(t),t)$	Endpoint position displacement of the refueling hose with respect to time t, where $0 < E(t) \leq L$.
$w(z,t)$	Displacement of the refueling hose at position z for time t, where $0 < z < E(t)$.

Manuscript received September 5, 2020; revised December 24, 2020 and April 14, 2021; accepted April 15, 2021. Date of publication May 25, 2021; date of current version February 10, 2022. Manuscript received in final form May 9, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 62073030 and Grant 61803109, in part by the Scientific Research Projects of Guangzhou Education Bureau under Grant 202032793, in part by the Guangdong Basic and Applied Basic Research Foundation under Grant 2019A1515110728, in part by the Science and Technology Planning Project of Guangzhou City under Grant 202102010398, in part by the National Research Foundation (NRF) of Korea under the auspices of the Ministry of Science and ICT, South Korea, under Grant NRF-2020R1A2B5B03096000, and in part by Beijing Top Discipline for Artificial Intelligent Science and Engineering, University of Science and Technology Beijing. Recommended by Associate Editor Y. Orlov. *(Corresponding author: Zhijia Zhao.)*

Zhijie Liu and Jinyuan Liang are with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China, also with the Institute of Artificial Intelligence, University of Science and Technology Beijing, Beijing 100083, China, and also with the Shunde Graduate School, University of Science and Technology Beijing, Foshan 528300, China (e-mail: liuzhijie2012@gmail.com; liangjinyuan_01@163.com).

Zhijia Zhao is with the School of Mechanical and Electrical Engineering, Guangzhou University, Guangzhou 510006, China (e-mail: zhjzhaoscut@163.com).

Mehmet Önder Efe is with the Department of Computer Engineering, Hacettepe University, 06800 Ankara, Turkey (e-mail: onderefe@gmail.com).

Keum-Shik Hong is with the School of Mechanical Engineering, Pusan National University, Busan 46241, Republic of Korea (e-mail: kshong@pusan.ac.kr).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCST.2021.3079275.

Digital Object Identifier 10.1109/TCST.2021.3079275

I. Introduction

WITH the increasing popularity of unmanned aerial vehicles in the military field, autonomous aerial refueling as an important means of endurance has come into being and aroused wide interests of many researchers over the last few years. Among them, the probe-and-drogue system with its unique advantages, such as lightweight and simple operation, occupies the mainstream position. Vibration and deformation may arise in the hose due to the complicated air environment and its structural characteristics. However, the excessive irregular vibration of the hose would make docking difficult. Moreover, excessive vibration may lead to disastrous consequences of premature of the hose and even the destruction of the whole system. Therefore, great attention should be paid to vibration control of the refueling hose system. Recently, many scholars have extensively studied the control design for flexible hose systems. However, the existing studies only focused on probe-and-drogue refueling hose (PDRH) modeled as lumped parameter systems, and these control schemes cannot be directly used for a flexible hose system modeled as distributed parameter systems (DPSs) in which partial differential equations (PDEs) need to be adopted.

In recent decades, much attention has been paid to infinite-dimensional DPS [1]–[3], and several control schemes have been proposed, such as reduced order model-based control [4], distributed control [5], and boundary control [6], [7].

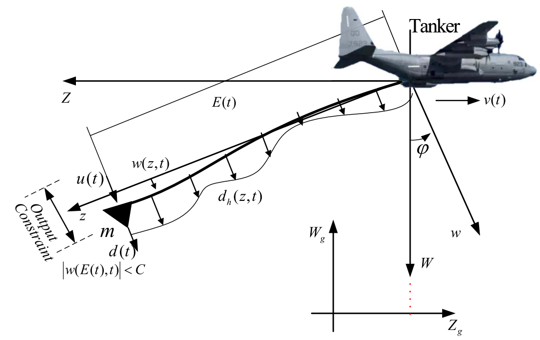

Fig. 1. Drawing of the PDRH system.

Compared to other methods mentioned above, the boundary control is nonintrusive in the process of sensing and actuation; thus, it is extensively adopted in the vibration suppression of DPS [8]–[10]. The input nonlinearities problem of flexible riser systems was handled through adaptive robust vibration control established in [11]. Iterative learning control schemes were introduced for coping with spatiotemporally varying disturbances for a flexible micro aerial vehicle in [12]. Especially, for flexible refueling hose systems, many boundary control schemes have been exploited to control the vibration based on PDEs recently. A novel dead-zone compensation method was developed for unknown dead-zone nonlinearity in [13]. In order to further research, the variable length of the flexible refueling hose was considered in our previous result [14]. A new boundary control law based on the back-stepping method was developed to regulate hoses vibration, where a Nussbaum function was adopted to solve the problem of input saturation. However, the abovementioned research about boundary control of refueling hose systems was restricted to suppressing vibrations or handling nonlinear constraints, the approaches of which were invalid when taking actuator failures [15] and output constraints [16] into account. The occurrence of actuator failure may bring additional uncertainty and destroy the stability of the system [17]. Moreover, performance degradation, even system corruption, may break out once the specified constraints are violated [18], [19]. Therefore, it is necessary to incorporate actuator failures and output constraints into flexible hose systems when designing the boundary controller.

In this brief, we investigate a PDRH system subjected to variable length, actuator fault, and output constraints. The major contributions of this brief compared to existing research are summarized as follows.

1) An adaptive fault-tolerant control scheme is developed to tackle partial effectiveness loss of the actuator.
2) The barrier Lyapunov function is applied to guarantee no violation of output constraints in the PDRH system.
3) With presented control schemes, the closed-loop system stability is demonstrated via the Lyapunov theory, and the system state can be guaranteed to converge to an arbitrarily small neighborhood of the origin.

II. Problem Formulation

Fig. 1 depicts the diagram of the PDRH system; three coordinate systems are established to describe the location of physical variables in the system. The inertial reference coordinate system is expressed by $Z_g - W_g$. As the tanker advances at speed $v(t)$, a coordinate system $Z - W$ is established to describe the relative position of the variables with the tanker as the origin. Also, a coordinate system $z - w$ based on the displacement of the hose $w(z,t)$ is established, the angle between it and the $Z - W$ coordinate system is φ, and the actuator $u(t)$ is installed on the drogue at location of $(E(t), t)$ on the z-axis.

A. Equations of Motion for the System

Remark 1: For partial differential operations, notations $(*)_z = ((\partial(*))/\partial z)$, $(*)_{zz} = ((\partial^2(*))/\partial z^2)$, $(*)_t = ((\partial(*))/\partial t)$, and $(*)_{tt} = ((\partial^2(*))/\partial t^2)$ are used throughout this brief.

Considering the following flexible hose system in [14] with boundary and distributed disturbances, the equations of motion are described as.

$$\rho[w_{tt}(z,t) + 2\dot{E}(t)w_{zt}(z,t) + \ddot{E}(t)w_z(z,t) + \dot{E}^2(t)w_{zz}(z,t)] = T_z(z,t)w_z(z,t) + T(z,t)w_{zz}(z,t) + Q(z,t) \quad (1)$$

where

$$T(z,t) = [m + \rho(E(t) - z)](g\sin\varphi - \ddot{E}(t) - \dot{v}(t)\cos\varphi) + f_{\text{drog}}(t)\cos\varphi + f_h(z,t) \quad (2)$$

and the auxiliary term $Q(z,t)$ is defined as

$$Q(z,t) = d_d(z,t) - f_n(z,t) + \rho(g\cos\varphi - \dot{v}(t)\sin\varphi) \quad (3)$$

with boundary conditions

$$m[2\dot{E}(t)w_{zt}(E(t),t) + \ddot{E}(t)w_z(E(t),t) + \dot{E}^2(t)w_{zz}(E(t),t)] + mw_{tt}(E(t),t) + m\dot{v}(t)\sin\varphi + T(E(t),t)w_z(E(t),t) - mg\cos\varphi = -f_{\text{drog}}(t)\sin\varphi + u(t) + d(t) \quad (4)$$

$$w(0,t) = 0. \quad (5)$$

Remark 2: It is noted that, for the differential operation of moving material, the concept of material derivative was introduced in [20], which is described as $D(\cdot)/Dt = \partial(\cdot)/\partial t + \dot{E}(t)\partial(\cdot)/\partial z$.

Remark 3: In the following derivation, we assume that the travel speed $v(t)$ of tanker is constant so that $\dot{v}(t) = 0$ and $\ddot{v}(t) = 0$.

An ideal actuator should be able to output the same amount of control as the input. However, in practice, loss of effectiveness may occur on the actuator, which will lead to the system cannot obtain ideal control input and even reduce stability. In this brief, we focus on one of the actuator failures, that is, partial loss of effectiveness, which can be modeled as

$$u(t) = \eta u_i(t) \quad (6)$$

where $u(t)$ represents the actual output of the actuator, $0 < \eta \le 1$, which represents the uncertain gains of the actuator during working, and $u_i(t)$ is the input value of actuator to be designed. The following two cases are discussed in this brief.

Case 1: $\eta = 1$, i.e., $u(t) = u_i(t)$, which is regarded as a failure-free actuator.

Case 2: $0 < \eta_0 \le \eta < 1$, which denotes that the actuator is suffering partial effectiveness loss, where η_0 is a certain constant, which implies the maximum degree of effectiveness loss. Specifically, $\eta = 80\%$ indicates a 20% loss of efficiency on the actuator.

Remark 4: The degree of effectiveness loss of the actuator can theoretically be any constant between 0 and 1. However, when the degree of effectiveness loss is very large, i.e., when η is very small, the system will become uncontrollable. At that point, the adaptive fault-tolerant control law designed in this brief can no longer meet the need to make the system stable. One of the solutions is to introduce redundant actuators into control design [21].

B. Preliminaries

In this part, we put forward some reasonable assumptions and necessary lemmas that are very helpful in carrying on the following study.

Assumption 1 [14]: In a practical system, since the energy of the disturbance is finite, therefore, we assume that $d(t)$ and $d_d(z,t)$ are bounded, which can be described as $|d(t)| \leq \overline{d}$ and $|d_d(z,t)| \leq \overline{d}_d$.

Assumption 2 [14]: It is assumed that the auxiliary term $Q(z,t)$ is bounded by a positive constant $Q_{\max}$, which can be expressed as $|Q(z,t)| \leq Q_{\max}, \forall (z,t) \in [0, E(t)] \times [0, \infty)$.

Assumption 3: We assume that $T(z,t)$, $T_z(z,t)$, and $T_t(z,t)$ are bounded by known constants, which satisfies the following inequalities:

$$0 < T_{\min} \leq T(z,t) \leq T_{\max} \tag{7}$$

$$-T_{z\max} \leq T_z(z,t) \leq -T_{z\min} < 0 \tag{8}$$

$$0 \leq T_{t\min} \leq T_t(z,t) \leq T_{t\max} \tag{9}$$

$\forall (z,t) \in [0, E(t)] \times [0, \infty)$.

Lemma 1 [22]: For a real function $h(t), t \in [t_0, \infty)$, the following inequality can be satisfied:

$$0 \leq |h(t)| - h(t) \tanh\left(\frac{h(t)}{\varsigma}\right) \leq \omega\varsigma \tag{10}$$

where $\varsigma > 0$ is a constant, and $\omega = 0.2785$.

Lemma 2 [23]: Let $\mathsf{V}(t) : [t_0, t) \to R$ with $t_0 \in (0, \infty)$; if $\dot{\mathsf{V}}(t) \leq -\lambda \mathsf{V}(t) + g(t)$, then

$$\mathsf{V}(t) \leq e^{-\lambda(t-t_0)}\mathsf{V}(t_0) + \int_{t_0}^{t} e^{-\lambda(t-\nu)} g(\nu) d\nu \tag{11}$$

where λ is a positive constant.

III. Model-Based Control Design Without Actuator Failure

In this section, we consider the actuator operates in the failure-free state, i.e., $u(t) = u_i(t)$. The control objectives are: 1) to abatement the PDRH vibration $w(z,t)$ and 2) to keep the boundary displacement $w(E(t),t)$ remaining in a given space in the situation where the length and speed of the flexible hose are time-varying.

The model-based control law $u(t)$ is proposed as follows:

$$\begin{aligned} u(t) = &-\psi(t)\left\{c_1 + \frac{m w(E(t),t)(Dw(E(t),t)/Dt)}{C^2 - w^2(E(t),t)}\right\}/S(t) \\ &- c_2\psi(t) - mg\cos\varphi + T(E(t),t)w_z(E(t),t) \\ &- \tanh\left(\frac{\psi(t)}{\varsigma}\right)\overline{d} + f_{\text{drog}}(t)\sin\varphi - kmw_{zt}(E(t),t) \\ &- km\dot{E}(t)w_{zz}(E(t),t) \end{aligned} \tag{12}$$

where c_1, c_2, and ς are positive control gains, C is the given boundary of $w(E(t),t)$, and $\psi(t)$ and $S(t)$ are designed as

$$\psi(t) = w_t(E(t),t) + \dot{E}(t)w_z(E(t),t) + kw_z(E(t),t) \tag{13}$$

$$S(t) = \ln\frac{2C^2}{C^2 - w^2(E(t),t)} \tag{14}$$

where k is a designed positive parameter.

Remark 5: From the definition of $S(t)$, we note that the boundary constraint C should satisfy $C > 0$ because the term $2C^2/(C^2 - w^2(E(t),t))$ must be positive. In addition, we know that the domain of definition of $S(t)$ is $|w(E(t),t)| < C$.

Remark 6: The term $-\tanh((\psi(t))/\varsigma)\overline{d}$ in the designed control input is continuous in its domain, such that the control input is also continuous. Symbolic function $-\text{sign}[\psi(t)]\overline{d}$ can also be employed to design the control law by replacing the hyperbolic tangent function mentioned above. In that condition, because of discontinuity of symbolic function, sliding motion may happen in control process [24], [25].

We choose the following Lyapunov candidate function for the PDRH system:

$$\mathsf{V}(t) = \mathsf{V}_1(t) + \mathsf{V}_2(t) + \mathsf{V}_3(t) \tag{15}$$

where $\mathsf{V}_1(t)$, $\mathsf{V}_2(t)$, and $\mathsf{V}_3(t)$ are the energy term, the barrier term, and the crossing term defined as follows:

$$\mathsf{V}_1(t) = \frac{\zeta}{2}\left[\int_0^{E(t)} \rho\left(\frac{Dw(z,t)}{Dt}\right)^2 + T(z,t)w_z^2(z,t)dz\right] \tag{16}$$

$$\mathsf{V}_2(t) = \frac{\zeta}{2} m\psi^2(t)S(t) \tag{17}$$

$$\mathsf{V}_3(t) = \varpi\int_0^{E(t)} \rho z\left(\frac{Dw(z,t)}{Dt}\right)w_z(z,t)dz \tag{18}$$

where ζ and ϖ are designed positive parameters.

Remark 7: In a real system, the actuator is installed at the end of the refueling hose, and the input of control value is mainly realized by active controllable drogue containing aerodynamic control surface [26]. The signals in the designed control law can be measured by sensors or can be calculated by the difference method. $w(E(t),t)$ and $w_z(E(t),t)$ can be obtained by a laser displacement and an inclinometer mounted on the tanker, respectively. $w_t(E(t),t)$, $w_{zz}(E(t),t)$, and $w_{zt}(E(t),t)$ can be obtained by the backward difference method. Due to the measurement noise of sensors, the actual acquired signals will have ineluctable errors, which will affect the control accuracy. Therefore, the allowable error range should be taken into account when selecting sensors.

Lemma 3: For the PDRH system described by (1)–(5), the Lyapunov function (15) is positive definite and bounded

$$0 \leq \varphi_1[\mathsf{V}_1(t) + \mathsf{V}_2(t)] \leq \mathsf{V}(t) \leq \varphi_2[\mathsf{V}_1(t) + \mathsf{V}_2(t)] \tag{19}$$

where φ_1 and φ_2 are positive constants.

Proof: Please see Appendix A.

Lemma 4: Consider the PDRH system described by (1)–(5) with the proposed control scheme (12). The time derivative of (15) is upper bounded as

$$\dot{\mathsf{V}}(t) \le -\lambda \mathsf{V}(t) + \delta \tag{20}$$

where λ and δ are positive constants.

Proof: Please see Appendix B.

Remark 8: The selection of parameter has a great influence on the system stability. On the basis of ensuring that the inequalities (40)–(45) hold true, the parameters are chosen for better control performance. The parameters contained in (40)–(45) should be designed in following steps. $\rho, L, T_{\max}, T_{\min}, T_{t\max}$, and $T_{z\max}$ are given or can be calculated in certain systems in practice. Equation (45) always holds true. Then, we design k, θ_1, θ_2, ϖ, and ζ such that (40), (41), (43), and (44) hold true, and ϖ and ζ should satisfy $0 < ((2\varpi\rho L)/(\zeta \min(\rho, T_{\min}))) < 1$ in order to make sure that the vibrations of the hose $w(z,t)$ are sufficiently small. Finally, c_1 is selected to guarantee (42) hold true. It is noted that the arbitrarily small vibration is guaranteed by the increase in control gains $c_i (i = 1, 2)$. However, very large control gains $c_i (i = 1, 2)$ would result in instability of the system. Hence, the control gains should be chosen prudently for satisfying the certain performance indicators in a real-world application.

Then, we can give the following theorem.

Theorem 1: For the PDRH system described by (1)–(5), under the action of the control scheme (12), provided that the intermediate parameters are appropriately selected to satisfy the inequalities (40)–(45), the following properties of the closed-loop system on the premise of the bounded initial conditions hold.

1) *Uniform Ultimate Boundedness (UUB) [27]:* System state will be guaranteed to converge to Ω_1, which is

$$\Omega_1 := \Big\{ w(z,t) \in R \,\Big|\, \lim_{t\to\infty} |w(z,t)| \le D_1, \quad \forall (z,t) \in [0, E(t)] \times [0, \infty) \Big\}$$

where $D_1 = ((2L\delta)/(\zeta T_{\min}\varphi_1\lambda))^{1/2}$ will be defined in Appendix C.

2) The boundary output of the PDRH is always within a certain space, i.e., the variable $w(E(t),t)$ of the system meets C, where C is the boundary constraint predefined in the above process.

Proof: Please see Appendix C.

The proposed model-based control can stabilize the PDRH system and keep the boundary displacement remaining in a given restricted boundary. However, due to the influence of the complex air environment, the actuator of the refueling hose system is prone to failure in operation. In order to ensure the control effect in such a situation, the adaptive fault-tolerant controller is designed subsequently.

IV. Adaptive Actuator Fault-Tolerant Control Design

In this part, we consider the case that actuator suffers partial effectiveness loss, i.e., $u(t) = \eta u_i(t), 0 < \eta_0 \le \eta < 1$. The control objective is to keep the boundary displacement $w(E(t),t)$ remaining in a given space even if the actuator is in the operation of a sudden partial effectiveness loss.

To motivate the following, we define:

$$p = \frac{1}{\eta}. \tag{21}$$

The estimation error between estimated value and true value of p is $\tilde{p} = \hat{p} - p$.

Then, we design the fault-tolerant control law as

$$u_i(t) = -\hat{p}\tau \tag{22}$$

where

$$\tau = -u(t) \tag{23}$$

and the adaptive control law

$$\dot{\hat{p}} = \tau\gamma\zeta\psi(t)S(t) - \kappa\hat{p} \tag{24}$$

where γ and κ are positive constants.

Consider the following Lyapunov candidate function as:

$$\mathsf{V}_a(t) = \mathsf{V}(t) + \frac{\eta}{2\gamma}\tilde{p}^2. \tag{25}$$

Lemma 5: For the PDRH system described by (1)–(5), the Lyapunov function (25) is positive definite and bounded

$$\begin{aligned} 0 \le \varphi_1\left[\mathsf{V}_1(t) + \mathsf{V}_2(t) + \frac{\eta}{2\gamma}\tilde{p}^2\right] &\le \mathsf{V}_a(t) \\ \le \varphi_2\left[\mathsf{V}_1(t) + \mathsf{V}_2(t) + \frac{\eta}{2\gamma}\tilde{p}^2\right]& \end{aligned} \tag{26}$$

where φ_1 and φ_2 are positive constants.

Proof: Please see Appendix D.

Lemma 6: Consider the PDRH system described by (1)–(5) with the proposed control scheme (22) and the adaptive law (24). The time derivative of (25) is upper bounded as

$$\dot{\mathsf{V}}_a(t) \le -\lambda_0 \mathsf{V}_a(t) + \delta_0 \tag{27}$$

where λ_0 and δ_0 are positive constants.

Proof: Please see Appendix E.

Then, we can give the following theorem.

Theorem 2: For the PDRH system described by (1)–(5), under the action of the control scheme (22)–(24), provided that the intermediate parameters are appropriately selected to satisfy the inequalities (40)–(45), the following properties of the closed-loop system on the premise of the bounded initial conditions hold.

1) *UUB [27]:* The system state will be guaranteed to converge to Ω_2, which is

$$\Omega_2 := \Big\{ w(z,t) \in R \,\Big|\, \lim_{t\to\infty} |w(z,t)| \le D_2, \quad \forall (z,t) \in [0, E(t)] \times [0, \infty) \Big\}$$

where $D_2 = ((2L\delta_0)/(\zeta T_{\min}\varphi_1\lambda_0))^{1/2}$ will be defined in Appendix F.

2) The boundary output of the flexible hose is always within a certain space, i.e., the variable $w(E(t),t)$ of the system meets $|w(E(t),t)| < C \forall t \in [0, \infty)$, where C is the output constraint given in the above process.

Proof: Please see Appendix F.

Remark 9: The control design procedure in this brief derives from Lyapunov's direct method. Therefore, compared with the controller in [14], which is based on the back-stepping

TABLE I

ARGUMENTS OF THE PDRH SYSTEM

Arguments	Definitions	Values
L	Maximum length of PDRH	$29m$
D_s	The diameter of PDRH	$0.067m$
ρ	The linear density of PDRH	$5.2kg/m$
ρ_{air}	The volume density of air	$1.29kg/m^3$
m	The weight of drogue	$39.5kg$
C_f	The coefficient of surface friction	0.005
C_d	The coefficient of pressure drag	0.45
C_{drog}	The coefficient of drag	0.43
D_{drog}	The diameter of drague	$0.61m$
$v(t)$	Flight speed of the tanker	$100km/h$
φ	Initial angle between W-axis and w-axis	$0.785rad$

method, the control strategies (12) and (22)–(24) and the design approach are more intuitive and easier to comprehend for engineers.

Remark 10: In the proof process of Theorems 1 and 2, the auxiliary term $\psi(t) = w_t(E(t),t) + \dot{E}(t)w_z(E(t),t) + kw_z(E(t),t)$ plays an important role in associating the Lyapunov function with the system boundary conditions. By taking derivative of the Lyapunov function and substituting the designed control law and the auxiliary term $\psi(t) = w_t(E(t),t) + \dot{E}(t)w_z(E(t),t) + kw_z(E(t),t)$ into it, we can obtain the following negative definite terms $-(\zeta/2)\rho(Dw(z,t)/Dt)^2$, $-(\zeta/2)T(z,t)w_z^2(z,t)$, $-(\zeta/2)m\psi^2(t)S(t)$, and $-(\eta/2\gamma)\tilde{p}^2$ (the last term is only used in Appendix B). Then, we can derive (31) and (52), which implies the boundedness of the Lyapunov functions $V(t)$ and $V_a(t)$, respectively. Furthermore, the conclusion stated in Theorems 1 and 2 can be obtained.

V. SIMULATIONS RESULTS

In this section, the availability of the control laws designed in Sections III and IV are verified by simulation results. Several approaches have been applied to discretization and numerical simulation of systems, such as the finite difference, the assumed mode method, the finite element method, and the Galerkin method, and good results can be obtained. In this brief, the PDRH considered in (1)–(5) are solved numerically by implementing a finite difference algorithm, and PD control (28) is also put forward for comparison. Simulations are made using MATLAB, and the system parameters are given in Table I. The time step size is given as 10^{-4} s, the space step size is given as $\Delta z = 0.02$ m, and the total simulation time is 50 s.

The boundary and distributed disturbances are set as $d(t) = 1.5\sin(0.5t) + 1.5\cos(0.5t)$ and $d_d(z,t) = 0.2 + 0.2\sin(0.5zt) + 0.2\sin(zt) + 0.2\sin(1.5zt)$. The resistance terms in system model can be expressed as

$$f_{\text{drog}} = \frac{1}{2}\rho_{\text{air}}v^2(t)C_{\text{drog}}\frac{\pi D_{\text{drog}}^2}{4}$$
$$f_n(z,t) = C_d\frac{1}{2}\rho_{\text{air}}v^2(t)\sin^2\varphi D_s$$
$$f_h(z,t) = C_f\frac{1}{2}\rho_{\text{air}}v^2(t)\cos^2\varphi\pi D_s.$$

The length the PDRH varies as $E(t) = 6 + 0.5t + 0.01t^2$ with initial conditions that $w(z,0) = 0.06z^2$ and $\dot{w}(z,0) = 0$. The initial condition of $\hat{p}$ is $\hat{p}(0) = 0$. The output constraint is set as $C = 0.6$ m, which can be seen in Figs.2(b) and 4(b) represented by blue dotted line. In order to study the algorithm designed in this brief, the verification is performed in following four cases.

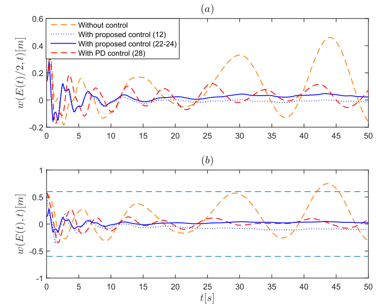

Fig. 2. Displacements of the PDRH when actuator works normally.

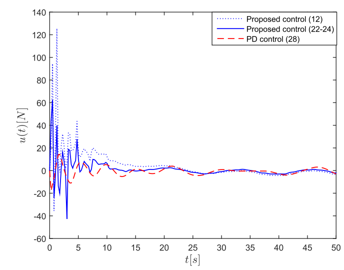

Fig. 3. Control inputs.

Case 1: Without control.

Case 2: The PD control is applied for the situation where the actuator works normally, which is proposed as follows:

$$u(t) = -k_p w(E(t),t) - k_d w_t(E(t),t) \tag{28}$$

where the control gains $k_p = 200$ and $k_d = 50$ are designed.

Case 3: In this case, the proposed control law (12) is utilized with control parameters $c_1 = 10, c_2 = 30$, and $k = 30$.

Case 4: In this case, we simulate the system performance by control schemes (22)–(24). We consider the situation where the actuator is subject to faults from 1s with the design parameters $\eta = 0.2, c_1 = 100, c_2 = 750, k = 28, \gamma = 1 \times 10^{-4}$, $\kappa = 0.073$, $\zeta = 1 \times 10^{-4}$, and $\zeta = 30$. For purpose of showing the superiority of fault-tolerant strategies (22)–(24), we also provide the control results of other two controllers in the same faulty situation of actuators.

Fig. 2 shows the displacements of the flexible hose when the actuator works normally for Case 1–4, and the control inputs are drawn in Fig. 3. Fig. 4 depicts the displacements of the flexible hose at the presence of an 80% effectiveness loss of the actuator for Case 1–4, whose control inputs are displayed in Fig. 5.

Fig. 2 compares the displacements at middle point $w(E(t)/2,t)$ and endpoint $w(E(t),t)$ for Case 1–4. In Fig. 2(b), it is shown that $w(E(t),t)$ continuously increases and eventually exceeds the blue dotted line without any control, which implies poor system performance. When the PD control is adopted, the vibration of the endpoint of the flexible hose is effectively suppressed; however, it causes oscillation at the middle point of the hose. Better control effect appears when we use proposed control schemes (12) and (22)–(24) instead. It can be seen that the displacements at both the middle point and the endpoint of the flexible hose continue

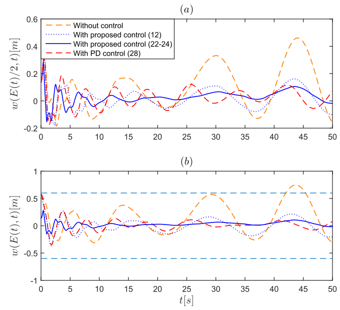

Fig. 4. Displacements of the PDRH when partial effectiveness loss occurs.

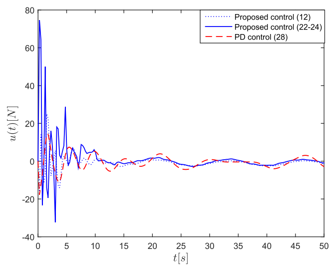

Fig. 5. Control inputs.

to converge to a small neighborhood around zero from the moment the control input is applied.

Fig. 4 depicts the different control results of different control strategies when the actuator suffers the partial loss of effectiveness. It can be informed that the control laws (12) and (28) cannot be used for acquiring desired control objectives any longer. Fortunately, the convergence of displacement of the flexible hose at both middle point and endpoint can still be guaranteed when the control schemes (22)–(24) are applied, which implies a satisfying control performance.

VI. Conclusion

In this study, we deal with adaptive fault-tolerant control of the variable length PDRH system influenced by external disturbances, actuator partial effectiveness loss, and output constraint. A model-based controller and an adaptive fault-tolerant controller are proposed for vibration control of the flexible hose during normal operation of the actuator and partial effectiveness loss of the actuator, respectively. In addition, the barrier Lyapunov function is employed to achieve system output boundary satisfaction. The closed-loop stability is demonstrated with the direct Lyapunov's method while ensuring the deflection eventually remaining in a small neighborhood of origin. Numerical simulations are finally performed to verify the availability and efficiency of the presented schemes. In future work, we aim to study intelligent methods to solve the vibration control of the PDRH system [28].

Appendix A
Proof of Lemma 3

We can obtain that $\mathsf{V}_3(t)$ satisfies the following inequality from its definition:

$$|\mathsf{V}_3(t)| \le \phi_1 \mathsf{V}_1(t) \tag{29}$$

where $\phi_1 = ((2\varpi\rho L)/(\zeta \min(\rho, T_{\min})))$. Therefore, we have

$$-\phi_1 \mathsf{V}_1(t) \le \mathsf{V}_3(t) \le \phi_1 \mathsf{V}_1(t). \tag{30}$$

Since ϖ satisfies $0 < \varpi < ((\zeta \min(\rho, T_{\min}))/(2\rho L))$, we obtain that $0 < \phi_1 < 1$, and

$$\phi_2 \mathsf{V}_1(t) \le \mathsf{V}_1(t) + \mathsf{V}_3(t) \le \phi_3 \mathsf{V}_1(t) \tag{31}$$

where $\phi_2 = 1 - \phi_1 > 0$ and $\phi_3 = 1 + \phi_1 > 1$. We further obtain

$$0 \le \varphi_1[\mathsf{V}_1(t) + \mathsf{V}_2(t)] \le \mathsf{V}(t) \le \varphi_2[\mathsf{V}_1(t) + \mathsf{V}_2(t)] \tag{32}$$

where $\varphi_1 = \min(\phi_2, 1) = \phi_2$ and $\varphi_2 = \max(\phi_3, 1) = \phi_3$.

Thus, (15) is positive definite and bounded.

Appendix B
Proof of Lemma 4

We take the material derivative of each part of $\mathsf{V}(t)$. Using (1) with boundary conditions (4), (5), and (13), we have

$$\begin{aligned}
\dot{\mathsf{V}}_1(t) &\le \frac{\zeta}{2\theta_1}\int_0^{E(t)}\left(\frac{Dw(z,t)}{Dt}\right)^2 dz \\
&\quad + \frac{\zeta}{2}\int_0^{E(t)} T_z(z,t)\dot{E}(t)w_z^2(z,t)dz + \frac{\theta_1\zeta}{2}\int_0^{E(t)} Q^2(z,t)dz \\
&\quad + \frac{\zeta}{2}\int_0^{E(t)} T_t(z,t)\big[w_z(z,t)\big]^2 dz + \frac{1}{2k}\zeta T(E(t),t)\psi^2(t) \\
&\quad + \left(\frac{1}{2}\zeta\rho - \frac{1}{2k}\zeta T(E(t),t)\right)\left(\frac{Dw(E(t),t)}{Dt}\right)^2 \\
&\quad + \left(\frac{1}{2}\zeta T(E(t),t) - \frac{k}{2}\zeta T(E(t),t)\right)[w_z(E(t),t)]^2 \\
&\quad - \frac{1}{2}[2\zeta T(0,t)\dot{E}(t) + \zeta\rho\dot{E}^2(t) + \zeta T(0,t)]\cdot[w_z(0,t)]^2.
\end{aligned} \tag{33}$$

The material derivative with respect to $\mathsf{V}_2(t)$ holds

$$\dot{\mathsf{V}}_2(t) = \zeta m\psi(t)\dot{\psi}(t)S(t) + \frac{\zeta}{2}m\psi^2(t)\dot{S}(t). \tag{34}$$

Substituting boundary conditions (4) and (5) and the proposed control law (12), we arrive at

$$\begin{aligned}
\dot{\mathsf{V}}_2(t) &= \zeta\psi(t)S(t)(-\psi(t)c_1/S(t) - c_2\psi(t)) \\
&\quad - \zeta\psi(t)S(t)\tanh\left(\frac{\psi(t)}{\varsigma}\right)\overline{d} + \zeta\psi(t)S(t)d(t).
\end{aligned} \tag{35}$$

According to Lemma 1, we have

$$\zeta\psi(t)S(t)d(t) - \zeta\psi(t)S(t)\tanh\left(\frac{\psi(t)}{\varsigma}\right)\overline{d} \le \zeta\omega\varsigma\overline{d}S(t) \tag{36}$$

where ω is the solution of $\omega = e^{-(1+\omega)}$.

Therefore, we can obtain that (34) becomes

$$\dot{\mathsf{V}}_2(t) \le -\zeta c_1\psi^2(t) - \zeta c_2\psi^2(t)S(t) + \imath \tag{37}$$

where $\imath = \zeta\omega\varsigma\overline{d}S(t)$.

Applying the governing equation (1) and boundary conditions (4) and (5), we arrive at

$$\begin{aligned}
\dot{\mathsf{V}}_3(t) &\le \frac{\varpi}{2}\int_0^{E(t)}(zT_z(z,t) - T(z,t) - \rho\dot{E}^2(t) + \theta_2 E(t)) \\
&\quad \times w_z^2(z,t)dz + \frac{\varpi E(t)}{2}(T(E(t),t) + \rho)w_z^2(E(t),t)
\end{aligned}$$

$$+\frac{\varpi L}{2\theta_2}\int_0^{E(t)} Q^2(z,t)dz - \frac{\varpi}{2}\rho\int_0^{E(t)}\left(\frac{Dw(z,t)}{Dt}\right)^2 \times dz + \varpi\rho E(t)\left(\frac{Dw(E(t),t)}{Dt}\right)^2. \quad (38)$$

Combining and appropriately rearranging the above inequalities, we obtain that $\dot{\mathsf{V}}(t)$ satisfies

$$\begin{aligned}\dot{\mathsf{V}}(t) &\le -\frac{1}{2}\int_0^{E(t)}(\varpi\rho\dot{E}^2(t) + \varpi T(z,t) - \varpi z T_z(z,t) - \varpi\theta_2 E(t) \\ &\quad - \zeta T_z(z,t)\dot{E}(t) - \zeta T_t(z,t))w_z^2(z,t)dz \\ &\quad -\frac{1}{2}\left(\varpi\rho - \frac{\zeta}{\theta_1}\right)\int_0^{E(t)}\left(\frac{Dw(z,t)}{Dt}\right)^2 dz \\ &\quad -\zeta c_2\psi^2(t)S(t) - \frac{1}{2}\left(2\zeta c_1 - \frac{1}{k}\zeta T(E(t),t)\right)\psi^2(t) \\ &\quad -\frac{1}{2}\left[\frac{1}{k}\zeta T(E(t),t) - \zeta\rho - 2\varpi\rho E(t)\right]\left(\frac{Dw(E(t),t)}{Dt}\right)^2 \\ &\quad -\frac{1}{2}[k\zeta T(E(t),t) - \zeta T(E(t),t) - \varpi T(E(t),t)E(t) \\ &\quad - \varpi\rho E(t)]w_z^2(E(t),t) \\ &\quad -\frac{1}{2}[2\zeta T(0,t)\dot{E}(t) + \zeta\rho\dot{E}^2(t) + \zeta T(0,t)]w_z^2(0,t) + \iota \\ &\quad +\frac{1}{2}\left(\theta_1\zeta + \frac{\varpi L}{\theta_2}\right)\int_0^{E(t)} Q^2(z,t)dz \end{aligned} \quad (39)$$

where the parameters are designed to meet the following conditions:

$$\varpi\rho\dot{E}(t)_{\min} + \varpi T_{\min} + \varpi L T_{z\min} + \zeta\dot{E}(t)_{\min}T_{z\min} - \varpi\theta_2 L - \zeta T_{t\max} \ge \varepsilon \quad (40)$$

$\forall(z,t)\in[0,L]\times[0,\infty)$, for some constants $\varepsilon > 0$, and the following inequalities hold true:

$$\varpi\rho - \frac{\zeta}{\theta_1} \ge 0 \quad (41)$$

$$2\zeta c_1 - \frac{1}{k}\zeta T_{\max} \ge 0 \quad (42)$$

$$\frac{1}{k}\zeta T_{\min} - \zeta\rho - 2L\varpi\rho \ge 0 \quad (43)$$

$$k\zeta T_{\min} - \zeta T_{\max} - \varpi L T_{\max} - L\varpi\rho \ge 0 \quad (44)$$

$$2\zeta T_{\min}\dot{E}(t)_{\min} + \zeta\rho\dot{E}^2(t)_{\min} + \zeta T_{\min} \ge 0. \quad (45)$$

Then, (39) can be simplified as

$$\dot{\mathsf{V}}(t) \le -\eta_1\frac{\zeta}{2}\int_0^{E(t)}\rho\left(\frac{Dw(z,t)}{Dt}\right)^2 dz - \eta_3\frac{\zeta}{2}m\psi^2(t)S(t) - \eta_2\frac{\zeta}{2}\int_0^{E(t)} T(z,t)w_z^2(z,t)dz + \delta \quad (46)$$

where $\eta_1 = (\varpi/\zeta) - (1/\rho\theta_1)$, $\eta_2 = (\varepsilon/\zeta T_{\max})$, $\eta_3 = (2c_2/m)$, and $\delta = (L/2)(\theta_1\zeta + (\varpi L/\theta_2))Q_{\max}^2 + \iota$.

We further have

$$\dot{\mathsf{V}}(t) \le -\varphi_3[\mathsf{V}_1(t) + \mathsf{V}_2(t)] + \delta \quad (47)$$

where $\varphi_3 = \min(\eta_1, \eta_2, \eta_3)$.

Combining (32) and (47), we obtain

$$\dot{\mathsf{V}}(t) \le -\lambda\mathsf{V}(t) + \delta \quad (48)$$

where $\lambda = \varphi_3/\varphi_2 > 0$.

Appendix C
Proof of Theorem 1

According to Lemma 3, we can obtain

$$\mathsf{V}(t) \le \mathsf{V}(0)e^{-\lambda t} + \varepsilon_0 \quad (49)$$

where $\varepsilon_0 = (\delta/\lambda)(1 - e^{-\lambda t})$.

We then have

$$\frac{\zeta T_{\min}}{2L}w^2(z,t) \le \frac{\zeta}{2}\int_0^{E(t)} T(z,t)w_z^2(z,t)dz \le \mathsf{V}_1(t) \le \frac{\mathsf{V}(t)}{\varphi_1}. \quad (50)$$

Then, we arrive at

$$|w(z,t)| \le \sqrt{\frac{2L}{\zeta T_{\min}\varphi_1}(\mathsf{V}(0)e^{-\lambda t} + \varepsilon_0)}. \quad (51)$$

From (32) and (49), we can inform that $\mathsf{V}(t)$ and $\mathsf{V}_1(t) + \mathsf{V}_2(t)$ are both positive and bounded, which indicates the boundedness of $\mathsf{V}_2(t)$. Therefore, we can infer that the boundary output $w(E(t),t)$ of the refueling hose is always in a certain set through proof by contradiction, which can be expressed as $\Theta_w := \{w(E(t),t) \in \mathbb{R} : |w(E(t),t)| < C\}$.

From (51), we know that $w(z,t)$ is also bounded. We can infer that $\lim_{t\to\infty}|w(z,t)| = ((2L\delta)/(\zeta T_{\min}\varphi_1\lambda))^{1/2} \le D_1 \in (0,\infty)$, which implies that the elastic vibration of the flexible refueling hose can be suppressed with appropriately selected parameters.

This completes the proof.

Appendix D
Proof of Lemma 5

Similar to Appendix A, we can obtain

$$\begin{aligned}0 \le \varphi_1\left[\mathsf{V}_1(t) + \mathsf{V}_2(t) + \frac{\eta}{2\gamma}\tilde{p}^2\right] &\le \mathsf{V}_a(t) \\ &\le \varphi_2\left[\mathsf{V}_1(t) + \mathsf{V}_2(t) + \frac{\eta}{2\gamma}\tilde{p}^2\right]\end{aligned} \quad (52)$$

where $\varphi_1 = \min(\phi_2, 1) = \phi_2$ and $\varphi_2 = \max(\phi_3, 1) = \phi_3$.

Thus, (25) is positive definite and bounded.

Appendix E
Proof of Lemma 6

Taking derivation of $\mathsf{V}_a(t)$ leads to

$$\dot{\mathsf{V}}_a(t) = \dot{\mathsf{V}}(t) + \frac{\eta}{\gamma}\tilde{p}\dot{\hat{p}}. \quad (53)$$

Compared to the material derivative of $\mathsf{V}(t)$ in the previous process, change occurs in $\dot{\mathsf{V}}_2(t)$ only. Substituting (21)–(23) into (34), we can obtain

$$\dot{\mathsf{V}}_2(t) = \zeta\psi(t)S(t)(-\psi(t)c_1/S(t) - c_2\psi(t)) - \frac{\eta\kappa}{2\gamma}\tilde{p}^2 + \frac{\eta\kappa}{2\gamma}p^2 - \zeta\psi(t)S(t)\tanh\left(\frac{\psi(t)}{\varsigma}\right)\overline{d} + \zeta\psi(t)S(t)d(t). \quad (54)$$

According to Lemma 1, we have

$$\zeta\psi(t)S(t)d(t) - \zeta\psi(t)S(t)\tanh\left(\frac{\psi(t)}{\varsigma}\right)\overline{d} \le \zeta\omega\varsigma\overline{d}S(t). \quad (55)$$

Therefore, $\dot{\mathsf{V}}_2(t)$ leads to

$$\dot{\mathsf{V}}_2(t) \leq -\zeta c_1 \psi^2(t) - \zeta c_2 \psi^2(t) S(t) - \frac{\eta\kappa}{2\gamma}\tilde{p}^2 + \iota + \frac{\eta\kappa}{2\gamma}p^2. \tag{56}$$

Similar to (39), substituting (33), (38), and (56) into (53), we arrive at

$$\dot{\mathsf{V}}_a(t) \leq -\lambda V(t) - \frac{\eta\kappa}{2\gamma}\tilde{p}^2 + \delta + \frac{\eta\kappa}{2\gamma}p^2. \tag{57}$$

We further obtain

$$\dot{\mathsf{V}}_a(t) \leq -\lambda_0 \mathsf{V}_a(t) + \delta_0 \tag{58}$$

where $\lambda_0 = \min(\lambda, \kappa)$, $\delta_0 = \delta + (\eta\kappa/2\gamma)p^2$.

Appendix F
Proof of Theorem 2

According to Lemma 3, we can obtain

$$\mathsf{V}_a(t) \leq \mathsf{V}_a(0)e^{-\lambda_0 t} + \varepsilon_1 \tag{59}$$

where $\varepsilon_1 = (\delta_0/\lambda_0)(1 - e^{-\lambda_0\ t})$.

Then, we have

$$\begin{aligned} \frac{\zeta T_{\min}}{2L} w^2(z,t) &\leq \frac{\zeta}{2}\int_0^{E(t)} T(z,t)[w_z(z,t)]^2 dz \leq \mathsf{V}_1(t) \\ &\leq \frac{\mathsf{V}(t)}{\varphi_1}. \end{aligned} \tag{60}$$

Then, we arrive at

$$|w(z,t)| \leq \sqrt{\frac{2L}{\zeta T_{\min}\varphi_1}(\mathsf{V}(0)e^{-\lambda_0 t} + \varepsilon_1)}. \tag{61}$$

Therefore, we can make a conclusion that the boundary output $w(E(t), t)$ of the refueling hose always remains in a certain set under the situation of actuator partial effectiveness loss through proof by contradiction, which can be expressed as $\Theta_w := \{w(E(t), t) \in \mathbb{R} : |w(E(t), t)| < C\}$. Furthermore, we can obtain that $\lim_{t\to\infty} |w(z, t)| = (2L\delta_0/(\zeta T_{\min}\varphi_1\lambda_0))^{1/2} \leq D_2 \in (0, \infty)$, which indicates that the elastic vibration of the refueling hose can be suppressed if the control parameters are appropriately chosen.

This completes the proof.

References

[1] J. Wang, S.-X. Tang, and M. Krstic, "Adaptive output-feedback control of torsional vibration in off-shore rotary oil drilling systems," *Automatica*, vol. 111, Jan. 2020, Art. no. 108640, doi: 10.1016/j.automatica.2019.108640.

[2] M. Krstic and A. Smyshlyaev, "Adaptive control of PDEs," *Annu. Rev. Control*, vol. 32, no. 2, pp. 149–160, 2008.

[3] J. Wang and M. Krstic, "Vibration suppression for coupled wave PDEs in deep-sea construction," *IEEE Trans. Control Syst. Technol.*, early access, Jul. 30, 2020, doi: 10.1109/TCST.2020.3009660.

[4] C. Sun, W. He, and J. Hong, "Neural network control of a flexible robotic manipulator using the lumped spring-mass model," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 47, no. 8, pp. 1863–1874, Aug. 2017.

[5] Y. Orlov, A. Pisano, and E. Usai, "Exponential stabilization of the uncertain wave equation via distributed dynamic input extension," *IEEE Trans. Autom. Control*, vol. 56, no. 1, pp. 212–217, Jan. 2011.

[6] W. He, T. Wang, X. He, L.-J. Yang, and O. Kaynak, "Dynamical modeling and boundary vibration control of a rigid-flexible wing system," *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 6, pp. 2711–2721, Dec. 2020.

[7] Y. Liu, Y. Fu, W. He, and Q. Hui, "Modeling and observer-based vibration control of a flexible spacecraft with external disturbances," *IEEE Trans. Ind. Electron.*, vol. 66, no. 11, pp. 8648–8658, Nov. 2019.

[8] W. He, X. He, M. Zou, and H. Li, "PDE model-based boundary control design for a flexible robotic manipulator with input backlash," *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 2, pp. 790–797, Mar. 2019.

[9] Z. Zhao, C. K. Ahn, and H.-X. Li, "Boundary antidisturbance control of a spatially nonlinear flexible string system," *IEEE Trans. Ind. Electron.*, vol. 67, no. 6, pp. 4846–4856, Jun. 2020, doi: 10.1109/TIE.2019.2931230.

[10] Z. Zhao, Z. Liu, W. He, K.-S. Hong, and H.-X. Li, "Boundary adaptive fault-tolerant control for a flexible timoshenko arm with backlash-like hysteresis," *Automatica*, vol. 130, Aug. 2021, Art no. 109690, doi: 10.1016/j.automatica.2021.109690.

[11] Z. Zhao, X. He, Z. Ren, and G. Wen, "Boundary adaptive robust control of a flexible riser system with input nonlinearities," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 49, no. 10, pp. 1971–1980, Oct. 2019.

[12] W. He, T. Meng, X. He, and C. Sun, "Iterative learning control for a flapping wing micro aerial vehicle under distributed disturbances," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1524–1535, Apr. 2019.

[13] L. Chang and Y. Jia, "Adaptive control of a hose and drogue system with input nonlinearities and partial state constraints," *Int. J. Control, Autom. Syst.*, vol. 17, no. 10, pp. 2508–2520, Oct. 2019.

[14] Z. Liu, J. Liu, and W. He, "Modeling and vibration control of a flexible aerial refueling hose with variable lengths and input constraint," *Automatica*, vol. 77, pp. 302–310, Mar. 2017.

[15] G. Tao, S. Chen, and S. M. Joshi, "An adaptive actuator failure compensation controller using output feedback," *IEEE Trans. Autom. Control*, vol. 47, no. 3, pp. 506–511, Mar. 2002.

[16] K. P. Tee, B. Ren, and S. S. Ge, "Control of nonlinear systems with time-varying output constraints," *Automatica*, vol. 47, no. 11, pp. 2511–2516, Nov. 2011.

[17] Z. Liu, Z. Han, Z. Zhao, and W. He, "Modeling and adaptive control for a spatial flexible spacecraft with unknown actuator failures," *Sci. China Inf. Sci.*, vol. 64, no. 5, May 2021, Art. no. 152208.

[18] W. Sun, Y. Liu, and H. Gao, "Constrained sampled-data ARC for a class of cascaded nonlinear systems with applications to motor-servo systems," *IEEE Trans. Ind. Informat.*, vol. 15, no. 2, pp. 766–776, Feb. 2019.

[19] Y.-J. Liu, S. Tong, C. L. P. Chen, and D.-J. Li, "Adaptive NN control using integral barrier Lyapunov functionals for uncertain nonlinear block-triangular constraint systems," *IEEE Trans. Cybern.*, vol. 47, no. 11, pp. 3747–3757, Nov. 2017.

[20] J. A. Schetz and A. E. Fuhs, *Fundamentals of Fluid Mechanics*. New York, NY, USA: Wiley, 1999.

[21] C. Wang, C. Wen, and Y. Lin, "Adaptive actuator failure compensation for a class of nonlinear systems with unknown control direction," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 385–392, Jan. 2017.

[22] M. M. Polycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica*, vol. 32, no. 3, pp. 423–427, Mar. 1996.

[23] J. S. A. Petros Ioannou, *Robust Adaptive Control*. New York, NY, USA: Dover, 2013.

[24] V. Acary, B. Brogliato, and Y. V. Orlov, "Chattering-free digital sliding-mode control with state observer and disturbance rejection," *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1087–1101, May 2012.

[25] V. Utkin, A. Poznyak, Y. V. Orlov, and A. Polyakov, *Road Map for Sliding Mode Control Design*. Cham, Switzerland: Springer, 2020.

[26] P. R. Thomas, U. Bhandari, S. Bullock, T. S. Richardson, and J. L. du Bois, "Advances in air to air refuelling," *Prog. Aerosp. Sci.*, vol. 71, pp. 14–35, Nov. 2014.

[27] I. G. Polushin, D. J. Hill, and A. L. Fradkov, "Strict quasipassivity and ultimate boundedness for nonlinear control systems," *IFAC Proc. Volumes*, vol. 31, no. 17, pp. 505–510, Jul. 1998.

[28] M. Cocetti, A. Serrani, and L. Zaccarian, "Linear output regulation with dynamic optimization for uncertain linear over-actuated systems," *Automatica*, vol. 97, pp. 214–225, Nov. 2018.