On Stabilization of Gradient-Based Training Strategies for Computationally Intelligent Systems

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***Abstract*—This paper develops a novel training methodology for computationally intelligent systems utilizing gradient information in parameter updating. The devised scheme uses the first-order dynamic model of the training procedure and applies the variable structure systems approach to control the training dynamics. This results in an optimal selection of the learning rate, which is continually updated as prescribed by the adopted strategy. The parameter update rule is then mixed with the conventional error backpropagation method in a weighted average. The paper presents an analysis of the imposed dynamics, which is the response of the training dynamics driven solely by the inputs designed by variable structure control approach. The analysis continues with the global stability proof of the mixed training methodology and the restrictions on the design parameters. The simulation studies presented are focused on the advantages of the proposed scheme with regards to the compensation of the adverse effects of the environmental disturbances and its capability to alleviate the inherently nonlinear behavior of the system under investigation. The performance of the scheme is compared with that of a conventional backpropagation. It is observed that the method presented is robust under noisy observations and time varying parameters due to the integration of gradient descent technique with variable structure systems methodology. In the application example studied, control of a two degrees of freedom direct-drive robotic manipulator is considered. A standard fuzzy system is chosen as the controller in which the adaptation is carried out only on the defuzzifier parameters.**

***Index Terms*—Fuzzy control, gradient descent, stable training, variable structure systems.**

NOMENCLATURE

f	Fuzzy system response.
ϕ	Generic parameter of fuzzy system.
ϕ^*	Optimal value of the generic parameter.
$\Delta\phi$	Change in parameter ϕ.
e	Observed output error.
d	Desired output.
J_r	Realization cost.
J_s	Parametric cost.
J_f	Logarithmic mapping on J_s.
η_ϕ	Learning rate for parameter ϕ.
T_s	Sampling interval of update dynamics.
s_ϕ	Switching function for parameter ϕ.
Q_ϕ	Gain of the switching scheme.
K_ϕ	Gain of the switching scheme.
ε	Boundary layer parameter.
N_ϕ	Backpropagated error value for parameter ϕ.
β	Scaling factor for parameter stabilizing law.
ζ_ϕ	Learning rate for cost minimizing law.
V_ϕ	Lyapunov function for parameter ϕ.
α_i	Weighting factor.
μ_{ij}	Membership function of ith rule's jth input.
c_{ij}	Center of membership function μ_{ij}.
u_j	jth input of computationally intelligent architecture.
a_{ij}, b_{ij}	Shape parameters of membership function μ_{ij}.
w	Vector of firing strengths.
w_n	Vector of normalized firing strengths.
$\theta_d(t)$	Desired state trajectory.
$\theta(t)$	Actual state trajectory.

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I. INTRODUCTION

STABILITY and robustness of the systems having adjustable parameters have been the primary focus of the field of systems science. The reason for this is mainly to maintain a desired behavior under the existence of factors influencing the performance and applicability adversely. In systems and control engineering practice, the applicability of an approach is directly relevant to the safeness of the approach. Strictly speaking, a method violating the stability requirements constitutes a potential danger from the safety point of view. The practice also imposes that the framework developed must meet the desired performance specifications, the achievement of which typically suffer from the environmental conditions, i.e., the existence of noise, time-varying parameters, or nonlinearities like saturation or time delays. Therefore, the concept of stability and robustness constitutes a central part particularly in the realm of control engineering. However, the rapid growth in science and technology has created complex systems having the capability of perceiving the environment and decision making. The innovations in data mining, data fusion, sensor technology, recognition technology, and fast microprocessors together with computationally intelligent system design have enabled the design and implementation of *expert-machine interaction*-based computation environments, which have the above mentioned capabilities.

Computational intelligence is a practical framework for solving complicated problems by utilizing expert knowledge, flexible architectures, and mathematical approaches determining the nature of artificial learning. The word *learning* in this context should be understood in the sense of a machine's capability of self-adaptation (parametric), self-organization, and self-diagnostics (architectural) in the face of varying environmental conditions without external intervention. This clearly implies a large spectrum in the domain of intelligence. In this respect, the degree of autonomy gains a crucial importance. This quantity is referred to as machine intelligence quotient (MIQ) in the related literature. Conceptually, the degree of intelligence is closely related to the design methodology followed. The limits of the intelligent behavior are determined by the flexibility of the architecture, the ability to realize the human expertise, laws of inference procedure, and the speed of learning. All of these titles are the main constituents of the research area called *computational intelligence*.

Fuzzy inference systems are the most popular constituent of the area of computationally intelligent systems since they are able to represent human expertise in the form of *IF antecedent THEN consequent* statements. In the design of fuzzy systems, the task to be achieved is modeled through the use of linguistic descriptions. Although the earliest work by Prof. Zadeh on fuzzy systems has not been paid as much attention as it deserved in early 1960s, since then the methodology has become a well-developed framework. The typical architectures of fuzzy inference systems are those introduced by Wang [1], [2], Takagi and Sugeno [3] and Jang [4]. In [1], a fuzzy system having Gaussian membership functions, product inference rule, and weighted average defuzzifier is constructed and has become the standard method in most applications. Takagi and Sugeno change the defuzzification procedure where dynamic systems are used in the defuzzification stage. The potential advantage of the Takagi–Sugeno fuzzy models is that under certain constraints, the stability of the system can be studied [5], [6]. Jang *et al.* [4] propose an adaptive neuro-fuzzy inference system in which a polynomial is used as the defuzzifier. This structure is commonly referred to as ANFIS in the related literature. The choice concerning the order of the polynomial and the variables to be used in the defuzzifier are left to the designer.

The approaches mentioned have widely been used for identification and control purposes [1]–[4], [6], [7]. As stated earlier, issues of stability and robustness are of crucial importance from safety and performance points of view. The implementation-oriented control engineering expert is, therefore, always in pursuit of a design that provides accurate tracking as well as insensitivity to environmental disturbances and structural uncertainties. At this point, it must be emphasized that these ambiguities can never be modeled accurately. When the designer tries to minimize the ambiguities by the use of a detailed model, then the design becomes so tedious that its cost increases dramatically. A suitable way of tackling with uncertainties without the use of complicated models is to introduce variable structure systems (VSS) theory based components into the design procedure.

Variable structure control (VSC) has successfully been applied to a wide variety of systems having uncertainties in the representative system models. The philosophy of the control strategy is simple, being based on two goals. First, the system is forced toward a desired dynamics. Second, the system is maintained on that differential geometry. In the literature, the former dynamics is named the reaching mode, while the latter is called the sliding mode. The control strategy borrows its name from the latter dynamic behavior and is called sliding mode control (SMC).

Earliest notion of SMC strategy was constructed on a second-order system in the late 1960s by Emelyanov [8]. The work stipulated that a special line could be defined on the phase plane such that any initial state vector can be driven toward the plane and then be maintained on it, while forcing the error dynamics toward the origin. Since then, the theory has greatly been improved and the sliding line has taken the form of a multidimensional surface, called the *sliding surface* and the function defining it is called the *switching function*.

Numerous contributions to VSS theory have been made during the last decade; some of them are as follows. Hung *et al.* [9] has reviewed the control strategy for linear and nonlinear systems. In [9], the switching schemes, putting the differential equations into canonical forms and generating simple SMC strategies are considered in detail. In [10] and [11], applications of SMC scheme to robotic manipulators are studied and the quality of the scheme is discussed from the point of robustness. One of the crucial points in SMC is the selection of the parameters of the sliding surface. Some studies devoted to the adaptive design of sliding surfaces have shown that the performance of a control system can be refined by interfacing it with an adaptation mechanism, which regularly redesigns the sliding surface [12], [13]. This eventually results in a robust control system. The performance of SMC scheme is proven to be satisfactory in the face of external disturbances and uncertainties in the system model representation. Another systematic examination of SMC approach is presented in [14]. In this reference, the practical aspects of SMC design are assessed for both continuous time and discrete time cases and a special consideration is given to the finite switching frequency, limited bandwidth actuators, and parasitic dynamics. In [15], the design of discrete time SMC is presented with particular emphasis on the system model uncertainties. Some studies consider the robustness property of VSS technique by equipping the system with computationally intelligent methods. In [7] and [16], fuzzy inference systems are proposed for SMC scheme. A standard fuzzy system is studied and the relevant robustness analyzes are carried out. Particularly, the work presented in [16] emphasizes that the robustness and stability properties of soft computing based control strategies can be analyzed through the use of SMC theory. It is shown in this reference that the approach is robust, i.e., it can compensate the deficiencies caused by poor modeling of plant dynamics and external disturbances.

The objective of this paper is to develop a training procedure for computationally intelligent architectures. The procedure enforces the adjustable parameters to settle down to a steady-state solution, while meeting the design specifications. This is achieved through an appropriate combination of error backpropagation (EBP) algorithm [17] with VSS philosophy. The early applications of VSS theory in training of computationally intelligent systems have considered the adjustment of the parameters of simple models like adaptive linear elements (ADALINE) [18]. The method presented in [18] is applied to the forward and inverse dynamics identification of a Kapitsa pendulum. A detailed analysis of VSS theory based training strategies for computationally intelligent systems can be found in [19]. The fundamental difference of the algorithm discussed in this paper is the fact that the derivation is based on the mixture of two different update values. Furthermore, the eventual form of the parameter update formula alleviates the handicaps of the gradient based training algorithms, which are widely used in the applications extending from speech processing to system identification and nonlinear control [20]–[30].

This paper is organized as follows. The second section briefly reviews the conventional EBP technique, which is responsible for achieving the desired performance specifications. The parameter stabilizing part of the training methodology is derived in the third section. The section starts with a continuous time representation of the EBP algorithm and continues with an explanation of how the VSS based training criterion and EBP-based training strategy are combined. In the fourth section, analysis of the imposed dynamics is presented. There it is shown that the desired dynamics and imposed dynamics are stable but structurally different. The fifth section gives the global stability proof of the mixed training strategy and discusses the constraints on the design parameters. In the sixth section, the standard fuzzy model is introduced and the application of the devised training strategy is discussed. The seventh section introduces a plant, which is to be controlled by using the architecture and the proposed learning algorithm. Simulation results are discussed in the eighth section and the conclusions are presented at the end of the paper.

II. Parameter Tuning with Error Backpropagation

In most applications of computationally intelligent systems, EBP method constitutes the central part of the learning. In this section, the technique is briefly reviewed for systems in which the outputs are differentiable with respect to the parameter of interest. The method has first been formulated for parameter adjustment in artificial neural networks by Rumelhart *et al.* [17] in 1980s. The approach has successfully been applied to a wide variety of optimization problems. Using the nomenclature, the algorithm can be stated as follows:

$$e = d - f(\phi, u) \tag{1}$$

$$J_r = \frac{1}{2}e^2 \tag{2}$$

$$\Delta\phi = -\eta_\phi \frac{\partial J_r}{\partial \phi}. \tag{3}$$

The observation error in (1) is used to minimize the realization cost in (2) by utilizing the rule described by (3), which is known as gradient descent or error backpropagation in the related literature

$$\Delta\phi = \eta_\phi e \frac{\partial f(\phi, u)}{\partial \phi}. \tag{4}$$

The minimization proceeds recursively as given in (4) for which the sensitivity derivative with respect to the generic parameter ϕ is needed. Since the update value in (4) entails the observation error e, the algorithm is quite sensitive to the noisy observations, which directly influence the value of the adjustable parameter and degrade the learning performance. The next section presents the derivation of a method capable of reducing the adverse effects of noise thereby increasing the robustness in this sense.

III. Parameter Tuning with Variable Structure Systems Approach

A continuous-time dynamic model of the parameter update rule prescribed by the EBP algorithm can be written as in

$$\dot{\Delta}\phi = -\frac{1}{T_s}\Delta\phi + \frac{\eta_\phi}{T_s}N_\phi. \tag{5}$$

The above model is composed of the sampling time denoted by T_s, the gradient-based nonscaled parameter change denoted by $N_\phi = e(\partial f(\phi, u)/\partial\phi)$ and a scaling factor denoted by η_ϕ for the selection of which a detailed analysis is presented in the subsequent discussion. Using Euler's first-order approximation for the derivative term, one obtains the following relation, which obviously validates the constructed model in (5) and which leads to the following representation:

$$\frac{\Delta\phi(k+1) - \Delta\phi(k)}{T_s} = -\frac{\Delta\phi(k)}{T_s} + \frac{1}{T_s}\eta_\phi N_\phi(k) \tag{6}$$

$$\Delta\phi(k+1) = \eta_\phi N_\phi(k). \tag{7}$$

By comparing (4) and (7), the equivalency between the continuous and discrete forms of the update dynamics is thus clarified. The synthesis of the parameter stabilizing component is based on the integration of the system in (5) with variable structure systems methodology. In the design of variable structure controllers, one method that can be followed is the reaching law approach [9]. For the use of this theory in the stabilization of the training dynamics, let us define the switching function as in (8) and its dynamics as in (9)

$$s_\phi = \Delta\phi \tag{8}$$

$$\dot{s}_\phi = -\frac{Q_\phi}{T_s}\tanh\left(\frac{s_\phi}{\varepsilon}\right) - \frac{K_\phi}{T_s}s_\phi \tag{9}$$

where Q_ϕ and K_ϕ are the gains and ε is the width of the boundary layer. In the derivations presented below, a key point is the fact that the system described by (5) is also driven by η_ϕ, which is known as learning rate in the related literature. Now we demonstrate that some special selection of this quantity leads to a rule that minimizes the magnitude of parametric displacement. With the quantity defined in (10), equating (9) and (5), and solving for $\Delta\phi$ yields the relation in

$$A_\phi = Q_\phi \tanh\left(\frac{\Delta\phi}{\varepsilon}\right) + K_\phi\Delta\phi \tag{10}$$

$$\Delta\phi = \eta_\phi N_\phi + A_\phi. \tag{11}$$

The values of the η_ϕ imposed by (11) might be seen as the desired values at first glance. However, this selection cancels out the backpropagated error value N_ϕ from (5), consequently, the update dynamics exactly behaves as that defined by the adopted switching function (9), which does not necessarily minimize the cost in (2). Therefore, the further analysis explores the restrictions on η_ϕ as well as the construction of the mixed training criterion.

Now we have a model described by (5) and an equality to be enforced and formulated by (11). If one chooses a positive definite Lyapunov function as given by (12), the time derivative of this function must be negative definite for stability of parameter change $(\Delta\phi)$ dynamics. Clearly, the stability in parameter change space implies the convergence in system parameters

$$V_\phi = \frac{1}{2}s_\phi^2 = \frac{1}{2}(\Delta\phi)^2 \tag{12}$$

$$\dot{V}_\phi = (\Delta\phi)(\dot{\Delta\phi}). \tag{13}$$

If (5) and (11) are substituted into (13), the constraint stated in (14) is obtained for stability in the Lyapunov sense

$$\eta_\phi^2 + \frac{1}{N_\phi}(A_\phi - \Delta\phi)\eta_\phi - \frac{1}{N_\phi^2}A_\phi\Delta\phi < 0. \tag{14}$$

Equation (14) can be rewritten in a more tractable form as follows:

$$\left(\eta_\phi + \frac{1}{N_\phi}A_\phi\right)\left(\eta_\phi - \frac{1}{N_\phi}\Delta\phi\right) < 0. \tag{15}$$

Since A_ϕ and $\Delta\phi$ have the same signs, the roots of (15) clearly have opposite signs. The expression on the left-hand side (LHS) assumes negative values between the roots. Therefore, in order to satisfy the inequality in (15), the learning rate must satisfy the constraint given in

$$0 < \eta_\phi < \min\left\{\left|\frac{1}{N_\phi}\Delta\phi\right|, \left|-\frac{1}{N_\phi}A_\phi\right|\right\}. \tag{16}$$

In order to preserve the compatibility between the traditional gradient-based approaches and the proposed approach, the interval of learning rate is restricted to positive values as described above. An appropriate selection of η_ϕ could be as follows:

$$\eta_\phi = \beta\min\left\{\left|\frac{1}{N_\phi}\Delta\phi\right|, \left|-\frac{1}{N_\phi}A_\phi\right|\right\}, \quad 0 < \beta < 1. \tag{17}$$

By substituting the learning rate formulated in (17) into the equality given in (11), the stabilizing component $\Delta\phi_{\mathrm{VSS}}$ of the parameter change formula is obtained as

$$\Delta\phi_{\mathrm{VSS}} = \beta\min(|\Delta\phi|, |A_\phi|)\,\mathrm{sgn}(N_\phi) + A_\phi \tag{18}$$

where $\Delta\phi$ on the right-hand side (RHS) is the final update value yet to be obtained. The law introduced in (18) minimizes the cost of stability, which is the Lyapunov function defined by (12). The question now reduces to the following. "Can this law minimize the cost defined by (2)?" The answer is obviously not because the stabilizing criteria in (18) is derived from the displacement of the parameter vector denoted by $\Delta\phi$, whereas the minimization of (2) is achieved when ϕ tends to ϕ^* regardless of the displacement. In order to minimize (2), the parameter change anticipated by gradient-based optimization technique, which is reviewed in the second section, should somehow be integrated into the final form of parameter update mechanism. As introduced in the second section, EBP algorithm evaluates a parameter change as given in

$$\Delta\phi_{\mathrm{EBP}} = \zeta_\phi N_\phi \tag{19}$$

where ζ_ϕ is the learning rate in the conventional sense. It is reasonable to expect that under certain constraints, a combination of the laws formulated in (18) and (19) in a weighted average will meet the objectives of both the parametric stabilization and the cost minimization, which means the fulfillment of the design specifications. The parameter update rule will then be as in

$$\Delta\phi = \frac{\alpha_1\Delta\phi_{\mathrm{VSS}} + \alpha_2\Delta\phi_{\mathrm{EBP}}}{\alpha_1 + \alpha_2}. \tag{20}$$

The parameter update formula given by (20) carries mixed displacement value containing both the parametric convergence, which is introduced by VSS part, and the cost minimization, which is due to the EBP technique. The balancing in this mixture is left to the designer by an appropriate selection of the positive weights α_1 and α_2. More explicitly, if $\alpha_1/\alpha_2 \gg 1$ the displacement given to the parameter ϕ has a strong tendency to maintain the current value of the parameter. However, when $\alpha_2/\alpha_1 \gg 1$ the mobility of the parameter vector increases and the important part of the parametric displacement value is dominated by the cost minimizing component. This clarifies how the selection of the weight parameters should be made in terms of the relative importance of the two subtasks.

IV. Analysis of the Imposed Dynamics

In the previous section, the mixed training algorithm is derived. This part analyzes the implications of the learning rate in (17) on the domain of parametric change space. If the learning rate in (17) is substituted into the dynamic model of (5), one ends up with the dynamics formulated in (21), which characterizes the behavior of the system driven solely by the learning rate in (17)

$$\dot{\Delta\phi} = -\frac{1}{T_s}\Delta\phi + \frac{\beta}{T_s}\min\{|\Delta\phi|, |A_\phi|\}\,\mathrm{sgn}(N_\phi). \tag{21}$$

In (21), two different cases can be of interest these are namely, $|\Delta\phi| < |A_\phi|$ or $|A_\phi| < |\Delta\phi|$. In the analysis presented below, the following two facts must be kept in mind.

Fact 1: $|x| = x\,\mathrm{sgn}(x)$ where $x \in \Re$.

Fact 2: $\mathrm{sgn}(x_1)\,\mathrm{sgn}(x_2) \le 1$, where $x_1, x_2 \in \Re$. For the first case, (21) becomes

$$\dot{\Delta\phi} = \frac{-1 + \beta\,\mathrm{sgn}(N_\phi)\,\mathrm{sgn}(\Delta\phi)}{T_s}\Delta\phi \le \frac{-1+\beta}{T_s}\Delta\phi. \tag{22}$$

Since $\beta < 1$, the imposed dynamics is globally stable. For the second case, (21) turns out to be as follows with the aid of Fact 3.

Fact 3: $A_\phi = Q_\phi\tanh(\Delta\phi/\varepsilon) + K_\phi\Delta\phi < (Q_\phi + K_\phi)\Delta\phi$ where $0 < \varepsilon \le 1$, which is the admissible interval for the boundary layer width. The lower bound on ε is due to the physical meaning, whereas the upper bound is due to the inequality given above

$$\begin{aligned}\dot{\Delta\phi} &= -\frac{1}{T_s}\Delta\phi + \frac{\beta}{T_s}A_\phi\,\mathrm{sgn}(N_\phi)\,\mathrm{sgn}(A_\phi)\\ &\le \frac{-1 + \beta(Q_\phi + K_\phi)}{T_s}\Delta\phi.\end{aligned} \tag{23}$$

If the term $\beta(Q_\phi + K_\phi)$ is constrained to be less than unity, the imposed dynamics becomes globally stable. The analysis presented in this section reveals that the imposed dynamics is somewhat different from the adopted switching function because of the constraints on the learning rate (η_ϕ) selection, nevertheless, the imposed dynamics is globally stable. In the next section, the overall stability proof of the training algorithm is discussed.

V. Extracting the Conditions for the Global Stability of the Mixed Training Dynamics

In this section, the global stability of the mixed training strategy is analyzed. For this purpose, a Lyapunov function given in (24) is defined. In (24), γ_ϕ is a positive constant and its properties are discussed at the end of the section

$$V_\phi = \frac{1}{2}(\Delta\phi)^2 + \frac{\gamma_\phi}{2}(N_\phi)^2. \tag{24}$$

The time derivative of the Lyapunov function is as given in

$$\dot{V}_\phi = \dot{\Delta}\phi\Delta\phi + \gamma_\phi \dot{N}_\phi N_\phi. \tag{25}$$

Since the analysis in this section concerns the stability of the mixed training strategy, the combined form of the learning algorithm, as given below, should be used in the formulation

$$\dot{V}_\phi = \left(-\frac{1}{T_s}\Delta\phi + \frac{\alpha_1\Delta\phi_{\mathrm{VSS}} + \alpha_2\Delta\phi_{\mathrm{EBP}}}{(\alpha_1+\alpha_2)T_s}\right)\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi. \tag{26}$$

If the $\Delta\phi_{\mathrm{VSS}}$ of (18) and $\Delta\phi_{\mathrm{EBP}}$ of (19) are substituted into (26), one obtains the following relation, which can assume two different forms due to the minimum operator:

$$\dot{V}_\phi = -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\min(|\Delta\phi|, |A_\phi|) \times \mathrm{sgn}(N_\phi)\Delta\phi + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi. \tag{27}$$

Case 1: $|\Delta\phi| < |A_\phi|$.

Since $|\Delta\phi| = \Delta\phi\,\mathrm{sgn}(\Delta\phi)$, (27) can be rewritten as follows:

$$\begin{aligned}\dot{V}_\phi &= -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\Delta\phi^2\,\mathrm{sgn}(\Delta\phi)\,\mathrm{sgn}(N_\phi) + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (28)\\ &= -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\Delta\phi^2\,\mathrm{sgn}(\Delta\phi)\,\mathrm{sgn}(N_\phi) + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}Q_\phi\tanh(\Delta\phi)\Delta\phi + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}K_\phi\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi. \qquad (29)\end{aligned}$$

Due to Section IV, Fact 3, the equality in (29) satisfies the following inequality:

$$\begin{aligned}\dot{V}_\phi &< -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\Delta\phi^2\,\mathrm{sgn}(\Delta\phi)\,\mathrm{sgn}(N_\phi) + \frac{\alpha_1 Q_\phi}{(\alpha_1+\alpha_2)T_s}\Delta\phi^2 + \frac{\alpha_1 K_\phi}{(\alpha_1+\alpha_2)T_s}\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (30)\\ &\le \left(\frac{(\beta+Q_\phi+K_\phi)\alpha_1}{(\alpha_1+\alpha_2)T_s} - \frac{1}{T_s}\right)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi. \qquad (31)\end{aligned}$$

If the RHS of (30) is rearranged with Section IV, the Fact 2, (31) is obtained. The inequality in (31) constitutes a time varying quantity, which is always larger than the quantity in (28). The further analysis for this case will proceed together with the result of the second case.

Case 2: $|A_\phi| < |\Delta\phi|$

$$\begin{aligned}\dot{V}_\phi &= -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}|A_\phi|\,\mathrm{sgn}(N_\phi)\Delta\phi + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (32)\\ &= -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi)A_\phi\Delta\phi + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (33)\\ &= -\frac{1}{T_s}\Delta\phi^2 + \left(\frac{\alpha_1\beta}{(\alpha_1+\alpha_2)T_s}\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi) + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}\right)A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (34)\\ &= -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}(\beta\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi)+1) \times A_\phi\Delta\phi + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi. \qquad (35)\end{aligned}$$

Fact 1: $A_\phi\Delta\phi < (Q_\phi + K_\phi)\Delta\phi^2$ and $A_\phi\Delta\phi > 0$.

Fact 2: $\min(\beta\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi) + 1) = 1 - \beta$ and $\max(\beta\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi) + 1) = 1 + \beta$ and $0 < \beta < 1$.

Due to the facts given above, the following rearrangements can be made:

$$\begin{aligned}\dot{V}_\phi &< -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}(\beta\,\mathrm{sgn}(A_\phi)\,\mathrm{sgn}(N_\phi)+1) \times (Q_\phi+K_\phi)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (36)\\ &\le -\frac{1}{T_s}\Delta\phi^2 + \frac{\alpha_1}{(\alpha_1+\alpha_2)T_s}(1+\beta)(Q_\phi+K_\phi)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (37)\\ &< -\frac{1}{T_s}\Delta\phi^2 + \frac{2\alpha_1}{(\alpha_1+\alpha_2)T_s}(Q_\phi+K_\phi)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (38)\\ &= \left(\frac{2\alpha_1(Q_\phi+K_\phi)}{(\alpha_1+\alpha_2)T_s} - \frac{1}{T_s}\right)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi\Delta\phi + \gamma_\phi\dot{N}_\phi N_\phi \qquad (39)\end{aligned}$$

$$< \left(\frac{2\alpha_1(\beta + Q_\phi + K_\phi)}{(\alpha_1+\alpha_2)T_s} - \frac{1}{T_s} \right) \Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s} \zeta_\phi N_\phi \Delta\phi + \gamma_\phi \dot{N}_\phi N_\phi. \quad (40)$$

If the negativity of the quantity on the RHS of the inequality (40) is ensured, the negativity of the quantity in (31) becomes trivial. Therefore, the two inequalities can be reduced to one inequality, which is given below. The global stability of the mixed training dynamics will clearly require the negativity of the quantity on the RHS of

$$\dot{V}_\phi < -\frac{C_\phi}{T_s}\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi \Delta\phi + \gamma_\phi \dot{N}_\phi N_\phi \quad (41)$$

where

$$C_\phi = 1 - \frac{2\alpha_1(\beta + Q_\phi + K_\phi)}{\alpha_1+\alpha_2}. \quad (42)$$

Set

$$\gamma_\phi = \frac{\sigma_\phi^2}{\sup_t |\dot{N}_\phi N_\phi|} \quad (43)$$

where σ_ϕ^2 is the least nonzero value of $\Delta\phi^2$ observed during a training course. It should be noted here that one may not know the numerical value of this number, but there exists such a number in the course of each training trial. With this value of γ_ϕ, (41) becomes as follows:

$$\dot{V}_\phi < -\frac{C_\phi}{T_s}\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi \Delta\phi + \frac{\sigma_\phi^2}{\sup_t |\dot{N}_\phi N_\phi|}\dot{N}_\phi N_\phi \quad (44)$$

$$< -\frac{C_\phi}{T_s}\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi \Delta\phi + \sigma_\phi^2 = B_\phi. \quad (45)$$

Inequality in (45) follows from the inequality

$$\frac{\dot{N}_\phi N_\phi}{\sup_t |\dot{N}_\phi N_\phi|} < 1. \quad (46)$$

Since $\sigma_\phi^2 \leq \Delta\phi^2$ for all $t \geq 0$

$$B_\phi \leq -\frac{C_\phi}{T_s}\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi N_\phi \Delta\phi + \Delta\phi^2 \quad (47)$$

$$< -\left(\frac{C_\phi}{T_s} - 1\right)\Delta\phi^2 + \frac{\alpha_2}{(\alpha_1+\alpha_2)T_s}\zeta_\phi |N_\phi||\Delta\phi|. \quad (48)$$

In order to ensure the negativeness of the RHS of (48), the following inequality must be satisfied:

$$\zeta_\phi < \frac{\alpha_1+\alpha_2}{\alpha_2}\frac{(C_\phi - T_s)|\Delta\phi|}{|N_\phi|}. \quad (49)$$

This selection of the learning rate for EBP part ensures the negative definiteness of the time derivative of the Lyapunov function in (24). It is clear that the parameter γ_ϕ exists, nonzero, nonnegative, and finite. These facts justify the particular chosen form of the Lyapunov function and the analysis proves that the solution given in (20) leads to the stable training of computationally intelligent systems.

It is clear that the derivation and the analysis presented impose some conditions on the design parameters. In the rest of this section these conditions are discussed.

1) Due to the requirement on the negative definiteness of the time derivative of the Lyapunov function, the following must be satisfied:

$$C_\phi = 1 - \frac{2\alpha_1(\beta + Q_\phi + K_\phi)}{\alpha_1+\alpha_2} > 0. \quad (50)$$

The selection for the learning rate ζ_ϕ imposes the following condition:

$$1 - \frac{2\alpha_1(\beta + Q_\phi + K_\phi)}{\alpha_1+\alpha_2} > T_s. \quad (51)$$

The inequality in (51) ensures the learning rate γ_ϕ to assume positive values. Since the condition in (51) includes the condition in (50), the constraint in (51) is one of the restrictions on the design parameters.

2) In order to ensure the stability of the imposed dynamics, which has already been analyzed in the fourth section, the following condition must hold true:

$$\beta(Q_\phi + K_\phi) < 1. \quad (52)$$

VI. Training of Fuzzy Inference Systems by the Developed Method

This section considers the standard fuzzy system approach introduced in [2] as the computationally intelligent architecture. The architecture utilized in this study uses bell-shaped membership functions as described by

$$\mu_{ij}(u_j) = \frac{1}{1 + \left|\frac{u_j - c_{ij}}{a_{ij}}\right|^{2b_{ij}}}. \quad (53)$$

In above, c_{ij} defines the center of the membership function, a_{ij} and b_{ij} characterize the slope and flatness of the function, respectively. The structure of the fuzzy system is illustrated in Fig. 1 for which the following type of a rule base structure is adopted:

IF u_1 is U_1 AND u_2 is U_2 AND $\cdots$ AND u_m is U_m
THEN $f = y_i$.

In the IF part of this representation, the lowercase variables denote the inputs and the uppercase variables stand for the fuzzy sets corresponding to the domain of each linguistic label. The THEN part is comprised of the prescribed decision in the form of a scalar number.

The overall realization performed by the system considered is given in (54), where weighted average defuzzifier is used with algebraic product aggregation method

$$f = \frac{\sum_{i=1}^R y_i \prod_{j=1}^m \mu_{ij}(u_j)}{\sum_{i=1}^R \prod_{j=1}^m \mu_{ij}(u_j)} = \sum_{i=1}^R y_i w_{ni} \quad (54)$$

where R is the number of rules contained in the rule base and m is the number of inputs. In (54), the vector of firing strengths de-

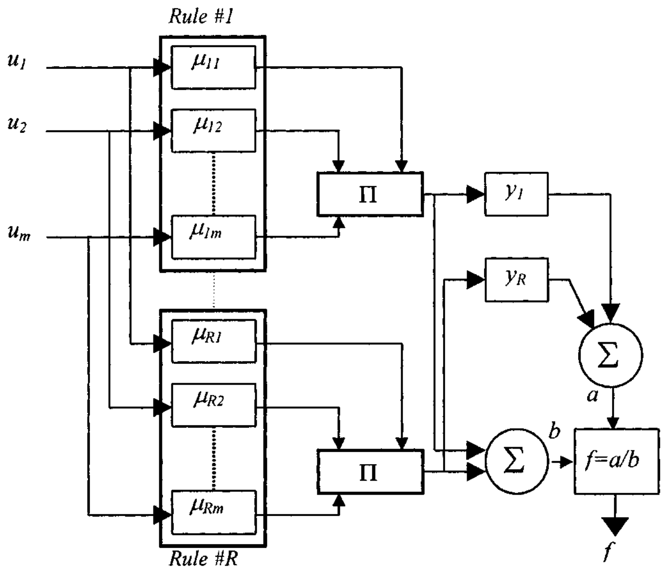

Fig. 1. Architecture of the standard fuzzy system.

noted by w is normalized and the resulting vector is represented by w_n

$$w_{ni} = \frac{\prod_{j=1}^{m} \mu_{ij}(u_j)}{\sum_{k=1}^{R} \prod_{j=1}^{m} \mu_{kj}(u_j)}. \tag{55}$$

With the definition given by (55) and the realization described by (54), the adjustable parameter set is selected as the y parameters of the defuzzifier. The backpropagated error measure can now be formulated as given by

$$N_{yi} = e\frac{\partial f(y_i, u)}{\partial y_i} = ew_{ni}. \tag{56}$$

By construction of the algorithm presented, the internal parameter A_{yi} is defined as follows:

$$A_{yi} = Q_{yi} \tanh\left(\frac{\Delta y_i}{\varepsilon}\right) + K_{yi}\Delta y_i. \tag{57}$$

The parameter ε that defines the boundary layer is selected as unity for all adjustable parameters and for all simulations presented in this study. The parameter stabilizing law defined in (18) imposes the update rule formulated in (58), whereas the cost minimizing update rule, which is the ordinary EBP method, predicts the necessary parameter change value as described by (59). The final form of the proposed update rule can now be formulated as a weighted average of these two values as described by (60)

$$\Delta y_{i\,\mathrm{VSS}} = \beta \min(|\Delta y_i|, |A_{yi}|)\,\mathrm{sgn}(N_{yi}) + A_{yi} \tag{58}$$

$$\Delta y_{i\,\mathrm{EBP}} = \zeta_{yi} N_{yi}. \tag{59}$$

In order to satisfy the conditions for the global stability, the learning rate for the EBP part is selected as follows:

$$\zeta_{yi} = 0.99\frac{\alpha_1 + \alpha_2}{\alpha_2}\frac{(C_{yi} - T_s)|\Delta y_i|}{|N_{yi}|} \tag{60}$$

$$\Delta y_i = \frac{\alpha_1 \Delta y_{i\,\mathrm{VSS}} + \alpha_2 \Delta y_{i\,\mathrm{EBP}}}{\alpha_1 + \alpha_2}. \tag{61}$$

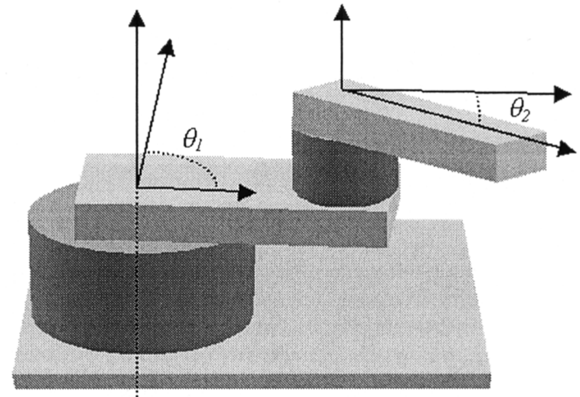

Fig. 2. View of the direct drive robotic manipulator.

VII. Plant Model

In the simulations, the dynamic model of a two degrees of freedom direct drive robotic manipulator (illustrated in Fig. 2) is used as the test bed. Since the dynamics of such a mechatronic system is modeled by nonlinear and coupled differential equations, precise output tracking becomes a difficult objective due to the strong interdependency between the variables involved. Besides, the ambiguities on the friction related dynamics in the plant model make the design much more complicated. Therefore, the methodology adopted must be intelligent in some sense.

The general form of robot dynamics is described by (62) where $M(\theta)$, $V(\theta, \dot{\theta})$, τ, and f_c stand for the state varying inertia matrix, vector of coriolis terms, applied torque inputs, and friction terms, respectively. The plant parameters are given in Table I in standard units

$$M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) = \tau - f_c. \tag{62}$$

If the angular positions and angular velocities are described as the state variables of the system, four coupled and first-order differential equations can define the model. In (63) and (64), the terms seen in (62) are given explicitly

$$M(\theta) = \begin{bmatrix} p_1 + 2p_3\cos(\theta_2) & p_2 + p_3\cos(\theta_2) \\ p_2 + p_3\cos(\theta_2) & p_2 \end{bmatrix} \tag{63}$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2)p_3\sin(\theta_2) \\ \dot{\theta}_1^2 p_3 \sin(\theta_2) \end{bmatrix}. \tag{64}$$

In above, $p_1 = 2.0857 + 0.0576M_p$, $p_2 = 0.1168 + 0.0576M_p$, and $p_3 = 0.1630 + 0.0862M_p$. Here M_p denotes the payload mass. The details of the plant model are presented in [31].

VIII. Simulation Results

In the simulations, the plant introduced in the Section VII is controlled by the standard fuzzy system considered in Section VI. The architecture of the control system is illustrated in Fig. 3.

The main objective of the design presented is to achieve precise state tracking together with small parameter update effort. This is achieved through a suitable combination of EBP algorithm and VSS methodology. During the simulations, c_{ij}, a_{ij}

TABLE I
MANIPULATOR PARAMETERS

Motor 1 Rotor Inertia	0.2670	I_1	Payload Mass	See text	M_p
Arm 1 Inertia	0.3340	I_2	Arm 1 Length	0.3590	L_1
Motor 2 Rotor Inertia	0.0075	I_3	Arm 2 Length	0.2400	L_2
Motor 2 Stator Inertia	0.0400	I_{3C}	Arm 1 Center of Gravity	0.1360	L_3
Arm 2 Inertia	0.0630	I_4	Arm 2 Center of Gravity	0.1020	L_4
Motor 1 Mass	73.000	M_1	Axis 1 Friction Bound	5.3000	f_{c1}
Arm 1 Mass	9.7800	M_2	Axis 2 Friction Bound	1.1000	f_{c2}
Motor 2 Mass	14.000	M_3	Torque Limit 1	245.00	
Arm 2 Mass	4.4500	M_4	Torque Limit 2	39.200	

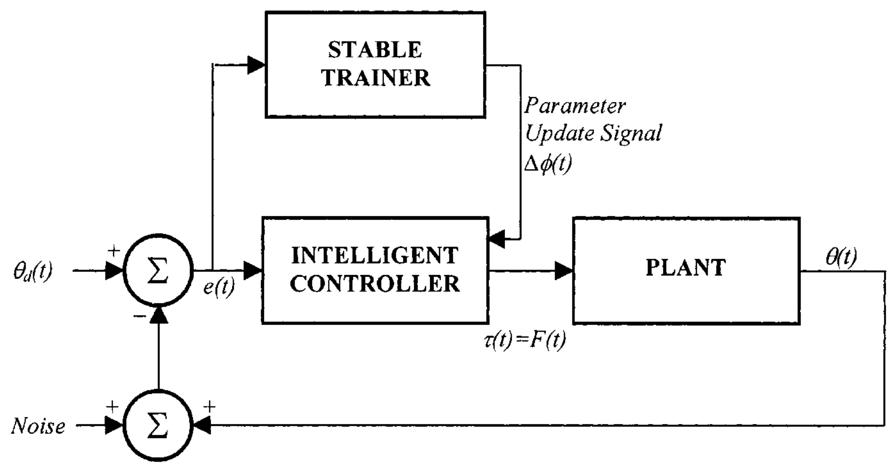

Fig. 3. Control of a plant using the proposed training method.

and b_{ij} parameters of the membership functions are kept constant and the adaptation is carried out on the y parameters of defuzzifier. The initial values of the membership functions are selected such that the region of interest is covered appropriately.

The reference angular position and velocity profiles used in all simulations are depicted in Fig. 4. The simulations are started with initial rest conditions.

In order to demonstrate the robustness property of the approach discussed, a payload of 2.5 kgs is regularly grasped and released by the robot. The time behavior of the payload conditions is demonstrated in Fig. 5. Another difficulty to be alleviated by the algorithm discussed is the observation noise. It is assumed that the encoders provide noisy measurements to the controller. The noise sequence is Gaussian distributed and has the same statistical properties for all four state variables, namely, each sequence has zero mean and variance equal to $33e{-}6$. The perturbing signal is illustrated in Fig. 6. It is expected that the stabilizing forces created on the adjustable design parameters will lead to the elimination of the adverse effects of the noisy observations, which excites the high frequency dynamics of the learning algorithm. Therefore, the results obtained will enable the designer to make a fair comparison between the pure gradient descent and the proposed combination especially in the sense of rejecting the high-frequency components entering into the training dynamics.

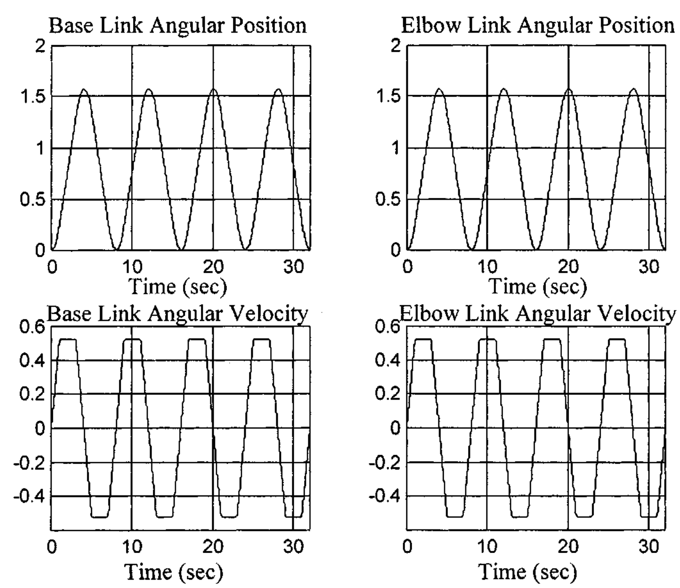

Fig. 4. Reference position and velocity trajectories.

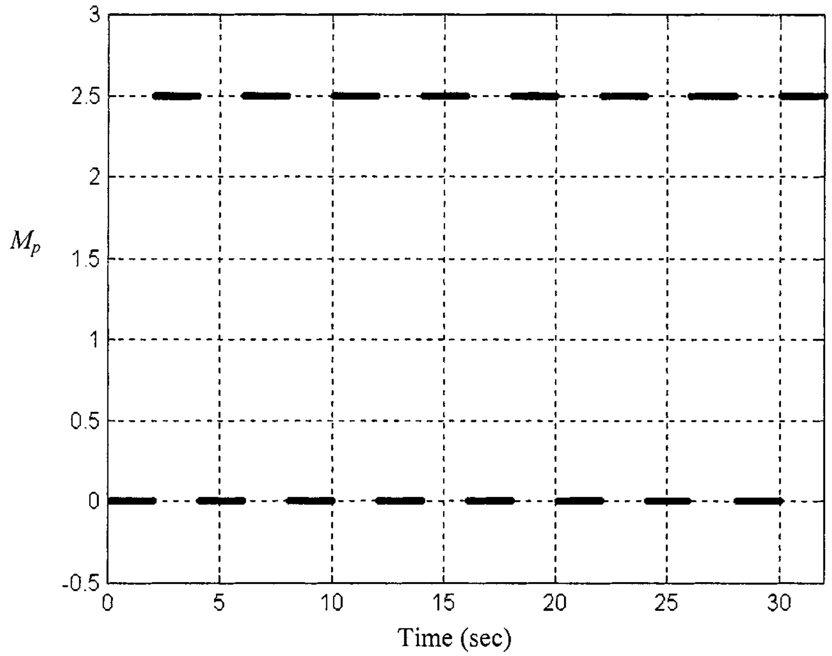

Fig. 5. Time behavior of the load mass.

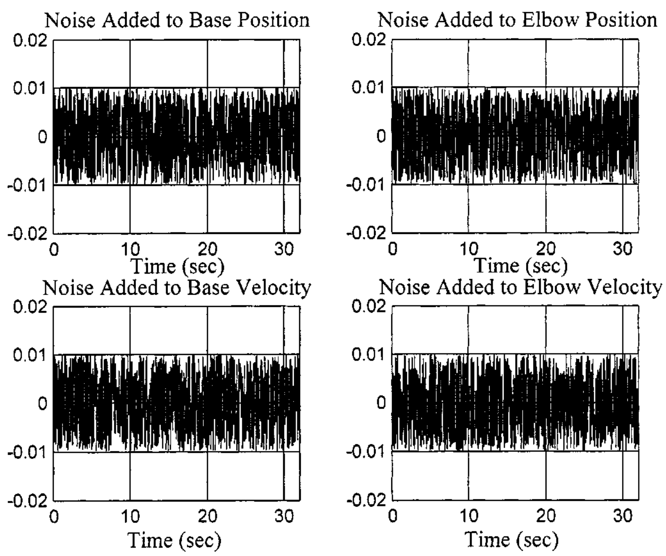

Fig. 6. The noise sequence added to the state variables.

In the training of the controller structures discussed in the paper, the squared sum of parametric changes is defined to be the cost of stability

$$J_s(t) = \sum_{\phi} (\Delta\phi(t))^2. \tag{65}$$

Since there are two controllers producing the necessary signals for each link, the summation in (65) is over the adjustable parameter set of both of the controllers.

The choice concerning the initial values of the membership function parameters is made by trial and error. Fuzzy quantization of the input variables is illustrated in Fig. 7. The state tracking errors and applied torque inputs are depicted in Figs. 8 and 9, respectively. It is evident from Fig. 8 that the proposed combination results in precise state tracking under the existence of disturbances stated above. Furthermore, Fig. 9 emphasizes that the control signals evaluated by the controllers lie within the limits of applicable control ranges. Therefore, the signal is directly applied to the manipulator without requiring saturation. The behavior of the total parametric cost described by (65) is figured out in the top row of Fig. 10. The upper left plot of Fig. 10 indicates that the cost in (65) reaches to very small values during the early phases of the simulation. This is due to the parameter stabilizing property of the approach discussed. As is discussed throughout the paper, the best convergence that can be attained by pure EBP technique reveals a marginally stable behavior, which is highly sensitive to the environmental disturbances. In order to visualize this behavior, the cost in (65) is mapped to another quantity defined below. Since $0 < J_s(t) \ll 1$, the mapping in (66) is a valid mapping

$$J_f(t) = \frac{1}{|\log(J_s(t))|}. \tag{66}$$

The function in (66) reveals the inverse power behavior of its argument. When (66) is plotted in polar coordinates, the globally stable behavior of the proposed technique and the marginally stable behavior of the traditional EBP technique can fairly be compared. This is because of the logarithmic nature of the radial direction, which makes the near origin activity more comprehensible. The bottom row of Fig. 10 illustrates the behavior of this function. In these subplots, time flows along the counterclockwise direction and one revolution corresponds to one period of the reference signal, i.e., 8 s period is 2π radians in these plots. Clearly, the existence of observation noise and the requirements of the problem in hand stimulate the unstable internal dynamics of EBP method. This is apparent from the right subplots of Fig. 10 in which the average magnitudes are increasing in time.

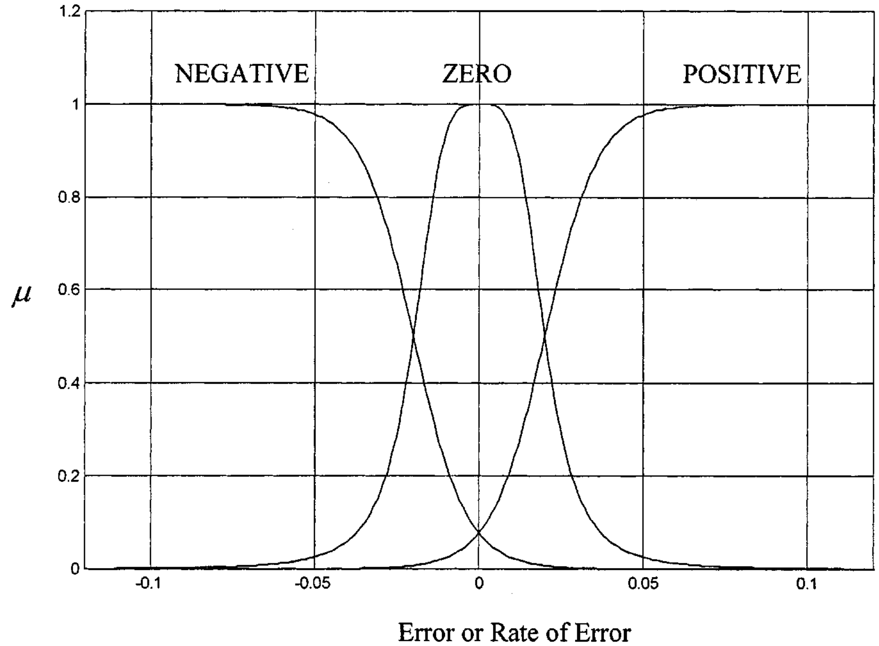

Fig. 7. Definitions of membership functions.

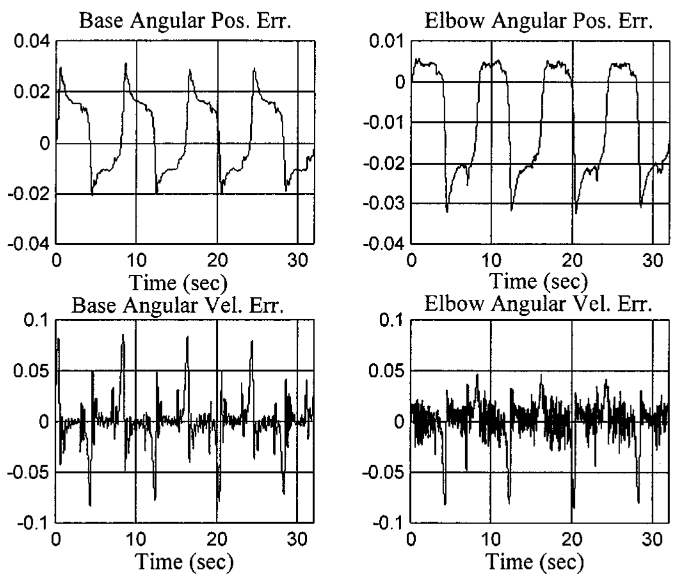

Fig. 8. State tracking errors with proposed method.

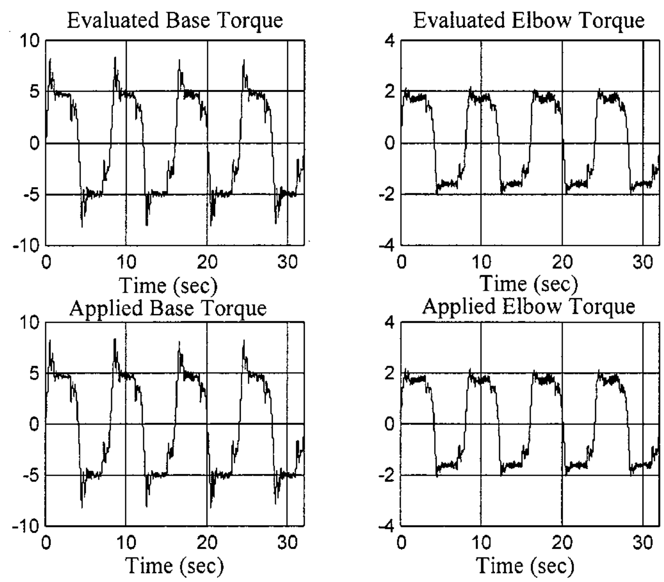

Fig. 9. State tracking errors in fuzzy control with proposed method.

For the use of the proposed algorithm, α_1 is set to 20 while α_2 is equal to 1. State tracking errors under the same conditions but only with EBP technique $(\alpha_1 = 0)$ is illustrated in Fig. 11. As is clearly seen, the results stipulate a divergent characteristic and the controller produces nonapplicable control signals. Due to the space limit, only the state tracking error behavior of the system under control is depicted. The simulation settings are tabulated in Table II in which it is apparent that the constraints stated in (51) and (52) are satisfied.

IX. Conclusion

One of the major problems in applications of gradient-based training strategies is the lack of stabilizing forces to prevent the adjustable parameters to grow unboundedly. This aspect of training without safety conditions constitutes a barrier between the theoretical developments and industrial applications whose prime concern is stability and robustness. The application examples utilizing the gradient information in training have, therefore, used the methods of computational intelligence, which are typically trained off-line with *a priori* data. In this paper, we propose a generalized method for creating stabilizing forces on the training dynamics of computationally intelligent systems, more specifically the typical forms of which are artificial neural networks, fuzzy inference systems, or systems capable of learning and generalizing knowledge.

The proposed method is based on the integration of EBP strategy with VSS technique to benefit from the robustness

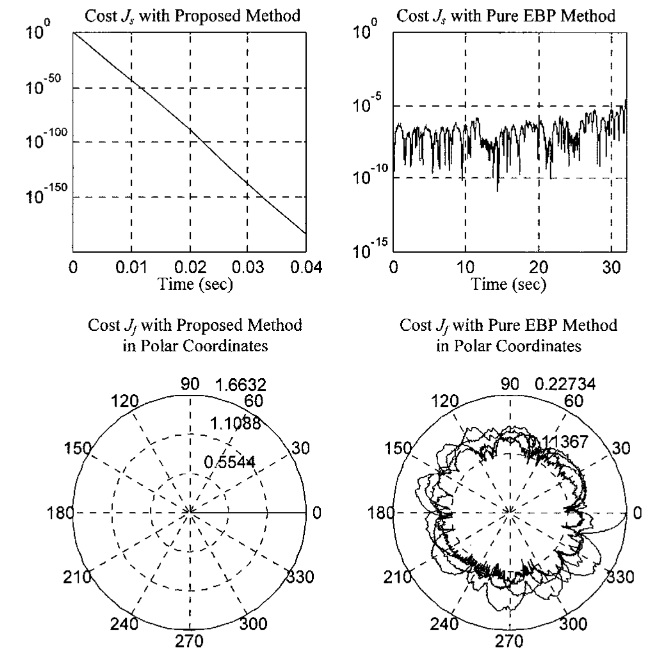

Fig. 10. Behavior of the parametric cost measures.

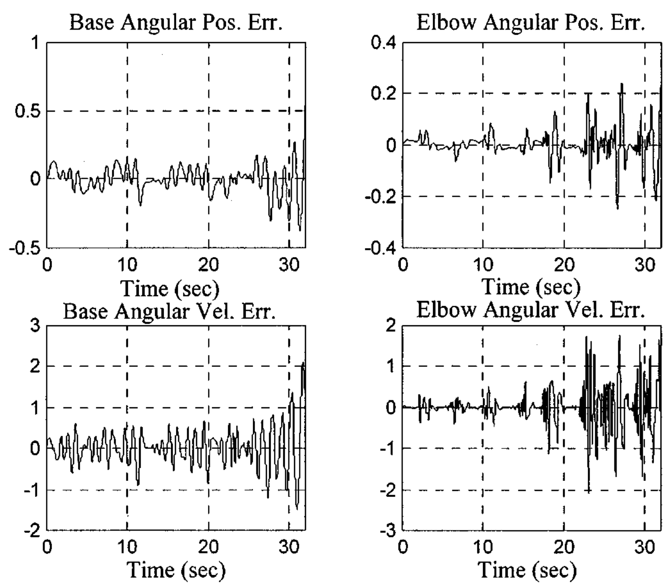

Fig. 11. State tracking errors with pure EBP technique.

property of VSS approach as well as the cost minimizing property of the EBP method. An analytical study of the conditions of stability in the parameter change space is presented. Since the extracted forms of the components $\Delta\phi_{\mathrm{VSS}}$ and $\Delta\phi_{\mathrm{EBP}}$ are mutually independent, the performance of the algorithm depends on the compatibility between the design objectives. More specifically, the applications entailing continuous evolution on the adjustable parameters can adversely be affected from the VSS based component as it enforces the parameters to converge.

Simulation studies carried out aim to compare the performance of the proposed scheme with that obtained with pure EBP technique. For this purpose, a standard fuzzy system is used as the computationally intelligent architecture and the adaptation is performed on the parameters of the defuzzifier. The task assigned to the fuzzy system is the control of a two degrees of freedom direct drive manipulator. Despite the representational simplicity of the plant, the existence of a considerable amount of observation noise corrupting the state variables and time-varying payload mass make the problem challenging for conventional learning schemes.

TABLE II
SIMULATION SETTINGS

T_s	1.0 msec.
β	0.1
α_1	See Sec. 7
α_2	1.0
Q	0.1
K	0.1
ε	1.0
#Rules	9 (for each link)
#FIS Inputs	2 (for each link)

The comparison strongly recommends the use of the algorithm for the applications requiring on-line tuning of the parameters, stability in the parametric displacement space and insensitivity to environmental disturbances.

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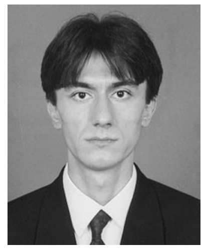

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