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# A Robust Identification Technique for Time-Varying ARMA Processes Based on Variable Structure Systems Theory

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## ABSTRACT

In this paper, a novel method for extracting the values of the coefficients of time-varying ARMA processes is proposed. The approach discussed assumes solely that the orders of the numerator and the denominator polynomials are known. The algorithm is demonstrated to be stable in the sense of Lyapunov, furthermore, it is shown in the paper that the evolution in the parameter space takes place in a finite volume. The proposed method is cost effective and is based on the variable structure systems theory, which is well known with its robustness to uncertainties. In the simulation example, the coefficients of a second order ARMA process is extracted by the use of the algorithm presented. The results confirm the prominent features of the proposed technique.

**Keywords:** ARMA processes, identification, parameter tuning, stable learning, variable structure systems.

## 1. INTRODUCTION

Identification of systems having uncertainties and impreciseness constitutes a central part in the practice of systems engineering. This fact is intimately related to the desire for constructing systems having a degree of autonomy enabling the system to operate in changing environmental conditions. One way of handling the difficulties stemming from the uncertainties is to utilize a suitable identification method to collect as much information as possible in an organized fashion. In the literature, most widely used approaches for system identification are based on Least Mean Squares (LMS),

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Recursive Least Squares (RLS), Gradient Descent (GD), Levenberg-Marquardt method (LM) or their variants [1–5]. The prime difficulties in implementing these algorithms are the necessity of costly hardware for data storage, high sensitivity to changes in the input signal, getting stuck to local minima or the need for matrix inversions at some intermediate stages. Depending on the nature of the problem in hand, the designer is expected to choose the most appropriate adjustment technique leading to high performance with low cost. Apparently, the design of such a parameter tuning scheme is a challenge even for the simple tasks. Among many alternatives existing in the literature for parameter tuning, once the structure of the ARMA process is chosen, the designer is faced to two performance measures, namely, the speed of adaptation and the accuracy of the realization after adaptation, which are intimately related to the adopted tuning strategy [3].

Another important feature of an adaptation mechanism is the robustness, which can be defined as the capability of compensating the uncertainties and the capability of keeping the cost measure at the lowest level under the presence of parameter variations in the process to be identified. If one defines the discrepancy between the output of a time-varying ARMA structure and that of an identifier ARMA structure as the error measure, the task of maintaining the zero output error with changing parameters clearly implies the need for a robustness in the tuning mechanism.

One way of designing a robust identification scheme is to utilize the Variable Structure Systems (VSS) theory in constructing the adaptation mechanism [6, 7]. The VSS theory is well-known with its robustness to uncertainties and the use of this theory introduces certain invariance properties in a predefined subspace of the state space, defined by the error and its several time derivatives. Various applications utilizing the VSS theory have appeared in the literature, which particularly focused on the robust control of nonlinear systems [6–9]. In order to understand the use of a technique of control engineering expertise for identification purposes, it is beneficial to dwell on what the framework prescribes in the field of control briefly.

For a system of order $l$, the decision dynamics in the state space is characterized by an ($l$-1)-dimensional hypersurface passing through the origin. Using the sign of a quantity describing the location of the hypersurface, two modes of motion can be created; namely the mode lasting until a hitting to the hypersurface occurs, and the mode on the hypersurface, which is called sliding surface. In the literature, the former is called *reaching mode* while the latter is named *sliding mode* and the control theory uses the term Sliding Mode Control (SMC) due to the latter dynamic behavior. An important property of the SMC design is that the trajectories in the phase space are all directed towards the sliding surface, furthermore, once the error vector starts lying on this hypersurface, it directly slides towards the origin because of the description of the hypersurface. The reason for this is not only the fact that the hypersurface itself is a location in the phase space, but it describes also a particular dynamics.

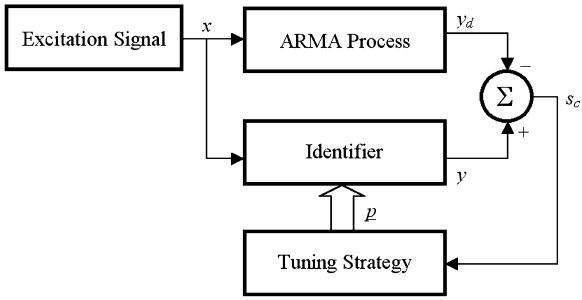


Fig. 1. Identification of a time-varying ARMA process.

In using the VSS theory for identifying ARMA processes, one should consider the diagram in Figure 1. The two structures are excited by the same signal ($x$) and the discrepancy between the produced outputs ($s_c$) is used as the error measure. This quantity is then processed in the tuning strategy and the corresponding parameter values ($\underline{p}$) are sent to the identifier and the next output is produced with the new parameter vector. Since the use of such identification techniques cover a wide spectrum, including nonlinear control, signal processing and biomedical applications, the method discussed in this paper is of substantial importance due to its computational advantages and robustness [5, 10–12].

In the second section, the structure of the ARMA process to be identified and the structure of the identifier are described and the proposed tuning strategy is analyzed in detail. The third section presents the simulations performed and the conclusions are presented at the end of the paper.

## 2. PROCESS DYNAMICS AND THE ADAPTATION STRATEGY

Consider the delay system described by (1).

$$y_d(t) = -\sum_{k=1}^{N} a_k y_d(t - kT) + \sum_{k=0}^{M} b_k x(t - kT) \tag{1}$$

where $M$ and $N$ define the delay depth in input ($x$) and output ($y_d$) respectively, and $T$ is the constant delay time parameter. The system above can be represented in $s$-domain as given in (2), and by direct substitution of $z = e^{sT}$, the $z$-domain equivalent is obtained as given in (3).

$$H_d(s) = \frac{Y_d(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k e^{-kTs}}{1 + \sum_{k=1}^{N} a_k e^{-kTs}} \tag{2}$$

$$H_d(z) = \frac{Y_d(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{3}$$

In above, the sampling period is assumed to be equal to $T$. The system given in (1) with $y_d(t) = 0$ for $t \leq 0$ and the transfer function representation in (2) produce the same outputs in response to the same excitation signals. When the response is compared with that obtained from (3) at integer multiples of delay time $T$, it should be obvious that the ARMA process in (3) is going to respond what is observed from the continuous time equivalent of (1).

The representations given in (1) through (3) describe the desired dynamics having some parameters denoted by $a_k$ and $b_k$ and these parameters are assumed to be unknown. The transfer function representation given in (3) can be expressed as a difference equation as given in (4), with $n$ being the discrete time index.

$$y_d[n] = -\sum_{k=1}^{N} a_k y_d[n-k] + \sum_{k=0}^{M} b_k x[n-k] \tag{4}$$

More compactly, the parameter vector and the regression vector can be described as in (5) and (6) respectively.

$$\underline{p}_d = [a_1 a_2 \ldots a_N \, b_0 \, b_1 \ldots b_M]^T \tag{5}$$

$$\begin{aligned} \underline{R}_d(t) = [&-y_d(t-T) - y_d(t-2T) \ldots \\ &- y_d(t-NT) \, x(t) \, x(t-T) \ldots x(t-MT)]^T \end{aligned} \tag{6}$$

The input/output relationship in (1) can be rewritten as follows:

$$y_d(t) = \underline{p}_d^T(t) \underline{R}_d(t) \tag{7}$$

We assume that the identifier has the same structure, which is described as $y(t) = \underline{p}^T(t) \underline{R}(t)$.

**Remark 2.1.** In order not to be in conflict with the physical reality, the designer must impose the following inequalities, the truth of which state that the parameters of the processes ($\underline{p}$ and $\underline{p}_d$), the time derivative of the regression vector ($\underline{R}$ and $\underline{R}_d$) and the time derivative of the desired output of the continuous processes ($y$ and $y_d$) remain bounded.

$$||\underline{p}|| = \sqrt{\underline{p}^T \underline{p}} \leq B_p \text{ and } ||\underline{p}_d|| = \sqrt{\underline{p}_d^T \underline{p}_d} \leq B_{pd} \tag{8}$$

$$\dot{\underline{R}} \leq B_{\dot{R}} \text{ and } \dot{\underline{R}}_d \leq B_{\dot{R}_d} \tag{9}$$

$$\dot{y} \leq B_{\dot{y}} \text{ and } \dot{y}_d \leq B_{\dot{y}_d} \tag{10}$$

**Remark 2.2.** In order to infer the true values of the unknown parameters ($\underline{p}_d$), the parameter vectors $\underline{p}$ and $\underline{p}_d$ must persistently be excited. Furthermore, if there exists a $\Gamma > 0$ such that $||\underline{R}(t)|| > \Gamma$ for $\forall t \geq 0$, it becomes apparent that that the regression vector follows a trajectory in $\Re^{M+N+1}$ such that its minimal distance to origin is greater than $\Gamma$.

**Corollary 2.3.** Since the system to be identified and the identifier has the same structure, from (6), $||\underline{R}(t)|| = \sqrt{\sum_{k=1}^{N} y(t-kT)^2 + \sum_{k=0}^{M} x(t-kT)^2} \geq \sqrt{\sum_{k=0}^{M} x(t-kT)^2}$. This obviously tells us that there exists a $\Gamma > 0$ and $||\underline{R}(t)|| > \Gamma$ for $\forall t \geq 0$ is satisfied unless the regression vector entries $x(t-kT) = 0$ for $0 \leq k \leq M+1$ and $\forall t \geq 0$.

In the following discussion, the value of $\mathrm{sgn}(s_c)$ is 1 if $s_c > 0$, $-1$ if $s_c < 0$ and zero if $s_c = 0$.

**Theorem 2.4.** The adaptation of process parameters as described in (11) enforces the process coefficients to values resulting in zero learning error level in one dimensional phase space, whose argument is defined as $s_c = y - y_d$.

$$\dot{\underline{p}} = -\frac{\underline{R}}{\underline{R}^T\underline{R}} K \,\mathrm{sgn}(s_c) \tag{11}$$

where, $K$ is a large positive constant satisfying (12).

$$K > B_p B_{\dot{R}} + B_{\dot{y}_d} \tag{12}$$

The adaptation mechanism in (11) drives an arbitrary initial value of $s_c$ to zero in finite time denoted by $t_h$ satisfying the inequality in (13).

$$t_h \leq \frac{|s_c(0)|}{K - (B_p B_{\dot{R}} + B_{\dot{y}_d})} \tag{13}$$

**Proof 2.4.** Consider the Lyapunov function candidate in (14). In order to reach to the zero learning error level ($s_c = 0$), the time derivative of (14) must be negative definite, which is given in (15).

$$V_c = \tfrac{1}{2} s_c^2 \tag{14}$$

$$\begin{aligned}
\dot{V}_c &= \dot{s}_c s_c \\
&= (\dot{y} - \dot{y}_d) s_c \\
&= (\dot{\underline{p}}^T \underline{R} + \underline{p}^T \dot{\underline{R}} - \dot{y}_d) s_c \\
&= -K \,\mathrm{sgn}(s_c) s_c + (\underline{p}^T \dot{\underline{R}} - \dot{y}_d) s_c \\
&= -K|s_c| + (\underline{p}^T \dot{\underline{R}} - \dot{y}_d) s_c \\
&\leq (-K + B_p B_{\dot{R}} + B_{\dot{y}_d}) |s_c|
\end{aligned} \tag{15}$$

It is apparent that the condition in (12) ensures the negative definiteness of the time derivative of the selected Lyapunov function.

If one evaluates $\dot{s}_c$ with the aid of (11), the expression in (16) is obtained. The solution to the differential equation in (16) can be given by (17).

$$\dot{s}_c = -K\,\mathrm{sgn}(s_c) + \underline{p}^T\dot{\underline{R}} - \dot{y}_d \tag{16}$$

$$s_c(t) - s_c(0) = -Kt\,\mathrm{sgn}(s_c(0)) + \int_0^t (\underline{p}^T(\sigma)\dot{\underline{R}}(\sigma) - \dot{y}_d(\sigma))d\sigma \tag{17}$$

At $t = t_h$, $s_c(t_h) = 0$;

$$-s_c(0) = -Kt_h\,\mathrm{sgn}(s_c(0)) + \int_0^{t_h} (\underline{p}^T(\sigma)\dot{\underline{R}}(\sigma) - \dot{y}_d(\sigma))d\sigma \tag{18}$$

By multiplying both sides of (18) by $-\mathrm{sgn}(s_c(0))$, one obtains (19).

$$\begin{aligned} |-s_c(0)| &= Kt_h + \left(\int_0^{t_h} (\underline{p}^T(\sigma)\dot{\underline{R}}(\sigma) - \dot{y}_d(\sigma))d\sigma\right)\mathrm{sgn}(s_c(0)) \\ &\geq Kt_h - (B_p B_{\dot{R}} + B_{\dot{y}d})t_h \end{aligned} \tag{19}$$

which implies hitting in finite time as described by the inequality in (13).

**Theorem 2.5.** If the identification scheme enters the sliding mode $s_c = 0$ and remains in it thereafter, then the parameters of the identifier, $\underline{p}$, evolve bounded.

**Proof 2.5**. In the sliding mode, ($s_c = 0$) and $\dot{s}_c = 0$. Based on this, the following derivation can be made.

$$\dot{s}_c = \dot{y} - \dot{y}_d \tag{20}$$

$$\dot{s}_c = \dot{\underline{p}}^T\underline{R} + \underline{p}^T\dot{\underline{R}} - \dot{y}_d = 0 \tag{21}$$

$$\underline{R}^T\dot{\underline{p}} = -\dot{\underline{R}}^T\underline{p} + \dot{y}_d \tag{22}$$

which requires the following,

$$\underline{R}^T\left(\dot{\underline{p}} + \frac{\underline{R}}{\underline{R}^T\underline{R}}\dot{\underline{R}}^T\underline{p} - \frac{\underline{R}}{\underline{R}^T\underline{R}}\dot{y}_d\right) = 0 \tag{23}$$

Since the entries of the vector $\underline{R}$ cannot be linearly dependent for all time [13], the equality in (23) imposes the following differential equation form in the sliding mode.

$$\dot{\underline{p}} = -\frac{\underline{R}\dot{\underline{R}}^T}{\underline{R}^T\underline{R}}\underline{p} + \frac{\underline{R}}{\underline{R}^T\underline{R}}\dot{y}_d \tag{24}$$

The solution to Equation (24) is as follows;

$$\underline{p}(t) = \Phi(t,0)\underline{p}(0) + \int_0^t \Phi(t,\sigma)\frac{\underline{R}(\sigma)}{\underline{R}(\sigma)^T\underline{R}(\sigma)}\dot{y}_d(\sigma)d\sigma \tag{25}$$

where,

$$\Phi(t,\alpha) = \exp\left(-\int_\alpha^t \frac{\underline{R}(\sigma)\dot{\underline{R}}(\sigma)^T}{\underline{R}(\sigma)^T\underline{R}(\sigma)}d\sigma\right) \tag{26}$$

Since $\Gamma \leq ||\underline{R}|| \leq B_R$, for the first term in (25), following relations can be induced.

$$\begin{aligned}
||\Phi(t,0)|| &= \left\|\exp\left(-\int_0^t \frac{\underline{R}(\sigma)\dot{\underline{R}}(\sigma)^T}{\underline{R}(\sigma)^T\underline{R}(\sigma)}d\sigma\right)\right\| \\
&= \left\|\exp\left(-\int_0^t \frac{\underline{R}(\sigma)}{\underline{R}(\sigma)^T\underline{R}(\sigma)}d\underline{R}(\sigma)^T\right)\right\| \\
&\leq \left\|\exp\left(\left|\left(-\int_0^t \frac{\underline{R}(\sigma)}{\underline{R}(\sigma)^T\underline{R}(\sigma)}d\underline{R}(\sigma)^T\right)\right|\right)\right\| \\
&\leq \left\|\exp\left(\int_0^t \frac{|\underline{R}(\sigma)|}{\underline{R}(\sigma)^T\underline{R}(\sigma)}d\underline{R}(\sigma)^T\right)\right\| \\
&< \left\|\exp\left(\Gamma^{-2}\int_0^t |\underline{R}(\sigma)|d\underline{R}(\sigma)^T\right)\right\| \\
&< \left\|\exp\left(\Gamma^{-2}B_R\int_0^t d\underline{R}(\sigma)^T\right)\right\| \\
&= ||\exp(\Gamma^{-2}B_R(\underline{R}(t)^T - \underline{R}(0)^T))|| \\
&\leq B_1
\end{aligned} \tag{27}$$

where $B_1$ is some positive constant. For the bound of the second term in (25), the analysis proceeds as given below.

$$\begin{aligned}
\left\|\int_0^t \Phi(t,\sigma)\frac{\underline{R}(\sigma)}{\underline{R}(\sigma)^T\underline{R}(\sigma)}\dot{y}_d(\sigma)d\sigma\right\| &< B_1\left\|\int_0^t \frac{\underline{R}(\sigma)}{\underline{R}(\sigma)^T\underline{R}(\sigma)}\dot{y}_d(\sigma)d\sigma\right\| \\
&< B_1\left\|\Gamma^{-2}\int_0^t \underline{R}(\sigma)\dot{y}_d(\sigma)d\sigma\right\| \\
&< B_1\Gamma^{-2}B_R\left\|\int_0^t \dot{y}_d(\sigma)d\sigma\right\| \\
&< B_1\Gamma^{-2}B_R\|y_d(t) - y_d(0)\| \\
&\leq B_2
\end{aligned} \tag{28}$$

where $B_2$ is some positive constant. Since the two components of the solution in (25) evolve bounded, the sum of them will trivially be bounded as given in (29).

$$||\underline{p}(t)|| < B_1 + B_2 \tag{29}$$

Note that in (8), we assumed that the parameters of the identifier, $\underline{p}$, are bounded. However, Theorem 2.5 states that once the identification scheme enters the sliding mode $s_c = 0$, the boundedness of $\underline{p}$ is guaranteed. That is to say that (8) is automatically satisfied.

## 3. SIMULATION STUDIES

In the simulations, we assume that the identification scheme observes the system output, the desired output and the regression vector at discrete time instants, i.e., the adaptation rule of (11) takes the following form.

$$\dot{\underline{p}}(t) = -\frac{\underline{R}(k)}{\underline{R}(k)^T \underline{R}(k)} K \operatorname{sgn}(s_c(k)) \quad \text{when } kT \le t \le (k+1)T \tag{30}$$

In the examples, an ARMA process structure described below is considered for identification. Apparently, the output of this system is what the identifier must realize by appropriately tuning its parameters.

$$y_d[n] = -\sum_{k=1}^{2} a_k y_d[n-k] + \sum_{k=0}^{2} b_k x[n-k] \tag{31}$$

It is assumed that the coefficients of the above structure are changing in time. In Table 1, the target values of the coefficients to be extracted are given. It should be clear from the table that there are four different ARMA structures, each one of which becomes active during a certain interval of 40 s. More explicitly, for the first 40 s, i.e., $r = 1$, the ARMA structure is characterized by the difference equation in (32), then a 5 s of transition phase takes place and during the following 40 s, i.e., $r = 2$, the

Table 1. Parameters of the ARMA process to be identified.

| Interval | Time (sec) | $b_0$ | $b_1$ | $b_2$ | $a_1$ | $a_2$ |
|---|---|---|---|---|---|---|
| $r = 1$ | $0 \le t \le 40$ | −1 | 0 | 1.2 | −0.1 | −0.72 |
| $r = 2$ | $45 \le t \le 85$ | 1 | −1.75 | −0.5 | 0.9 | 0.2 |
| $r = 3$ | $90 \le t \le 130$ | −1 | 0.65 | −0.8 | 0 | 0 |
| $r = 4$ | $135 \le t \le 160$ | 1 | 1.5 | 0.8 | 0.3 | 0.6 |

process is changed smoothly to that in (33). In (34) and (35), the structures activated in the next two intervals are described.

$$y_{d1}[n] = 0.1y_{d1}[n-1] + 0.72y_{d1}[n-2] - x[n] + 1.2x[n-2] \tag{32}$$

$$y_{d2}[n] = 0.9y_{d2}[n-1] - 0.2y_{d2}[n-2] + x[n] - 1.75x[n-1] - 0.5x[n-2] \tag{33}$$

$$y_{d3}[n] = -x[n] + 0.65x[n-1] - 0.8x[n-2] \tag{34}$$

$$y_{d4}[n] = -0.3y_{d4}[n-1] - 0.6y_{d4}[n-2] + x[n] + 1.5x[n-1] + 0.8x[n-2] \tag{35}$$

The change of structures between the successive intervals is performed by suitably combining the all four ARMA structures. For instance, the transition between the time instants 40 and 45 s is illustrated in Figure 2. A weight is gradually activating the structure in (33) by multiplying its output with $w$ and the effect of the structure in (32) is decreasing since its multiplier is $1 - w$. This enables us to observe a soft switching between different ARMA processes. In the transition regions, the mixed process possesses fourth order numerator and denominator. The corresponding coefficients can be evaluated as given in (36) through (44). The second subscripts are used to specify the interval number.

$$\tilde{b}_0 = (1-w)b_{0,r} + wb_{0,r+1} \tag{36}$$

$$\tilde{b}_1 = (1-w)(b_{0,r}a_{1,r+1} + b_{1,r}) + w(b_{0,r+1}a_{1,r} + b_{1,r+1}) \tag{37}$$

$$\begin{aligned}\tilde{b}_2 = {} & (1-w)(b_{1,r}a_{1,r+1} + b_{0,r}a_{2,r+1} + a_{2,r}) \\ & + w(b_{1,r+1}a_{1,r} + b_{0,r+1}a_{2,r} + a_{2,r+1})\end{aligned} \tag{38}$$

$$\tilde{b}_3 = a_{1,r}a_{2,r+1} + a_{1,r+1}a_{2,r} \tag{39}$$

$$\tilde{b}_4 = 1 \tag{40}$$

$$\tilde{a}_1 = a_{1,r} + a_{1,r+1} \tag{41}$$

$$\tilde{a}_2 = a_{2,r} + a_{2,r+1} + a_{1,r}a_{1,r+1} \tag{42}$$

$$\tilde{a}_3 = a_{1,r}a_{2,r+1} + a_{2,r}a_{1,r+1} \tag{43}$$

$$\tilde{a}_4 = a_{2,r}a_{2,r+1} \tag{44}$$

In the example considered a Gaussian distributed random signal having zero mean and variance equal to unity is used to excite the process to be identified and the identifier. Furthermore, the quantity $\sqrt{\sum_{k=0}^{2} x(t-kT)^2} \geq 0.0292$ for $\forall t \geq 0$, which implies that there exists a $\Gamma > 0$ satisfying $\Gamma < 0.0292$. The aim is to match the two output signals in time domain. For this purpose, the error ($s_c$) is evaluated and used in the adjustment mechanism of (30). The sampling rate ($T$) has been set to 1 ms, and the uncertainty bound parameter $K$ has been set to 100. Initially, the parameters of the identifier are all set to zero.

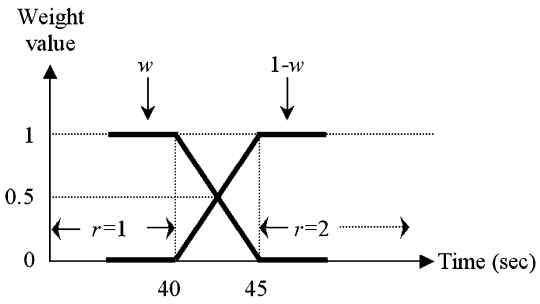


Fig. 2. The transition between intervals $r = 1$ and $r = 2$.

It should here be noted that the uncertainty bound parameter denoted by $K$ is a multiplier of the dynamic adaptation law of (30). Apparently, the small values of $K$ will decrease the speed of learning and large values will increase. However, since $K$ is a parameter describing the mobility of the parameters, its extreme values can cause instabilities. Unlike its lower bound described in (12), the upper bound is determined by the physical constraints and the environmental conditions.

The extracted values of the parameters are depicted in Figure 3. If one compares the values estimated by the identifier with those given in Table 1, it is seen that the correct values are found very quickly and very accurately. Especially the during transition intervals [40 s, 45 s], [85 s, 90 s] and [130 s, 135 s], the process to be identified becomes a fourth order one, which is characterized by (36)–(44), and the strategy proposed quickly converges when the order of the structure becomes equal to that of the identifier. In Figure 4, the pole-zero plots of the four different structures are illustrated. It can directly be inferred from this figure that the system under investigation is changing its characteristics radically, and this constitutes a challenge for most identification schemes.

An interesting observation on the results seen on Figure 3 is the emergence of ringing at different magnitudes. Since the rule determining the nature of parametric evolution (refer to (11)) is tightly dependent upon the sign of a quantity, which is very close to zero, such instant fluctuations are inherently introduced. However, the magnitudes of them are relevant to the unknown dynamics of the process under investigation. The extreme case is to include an integrator in the dynamics, i.e., a pole at $z = 1$. In the first trial, i.e., $r = 1$, the system has a real pole close to unity and a visible amount of ringing observed during its identification as the output of the system is influenced by the excitation comparably quicker than the remaining three dynamics. Comparing the pole locations of the processes, one can visualize that the ringing would be smaller for $r = 2$ and much smaller for $r = 3$ and 4. However, from a practical point of view, the operator may not have the prior knowledge to manipulate

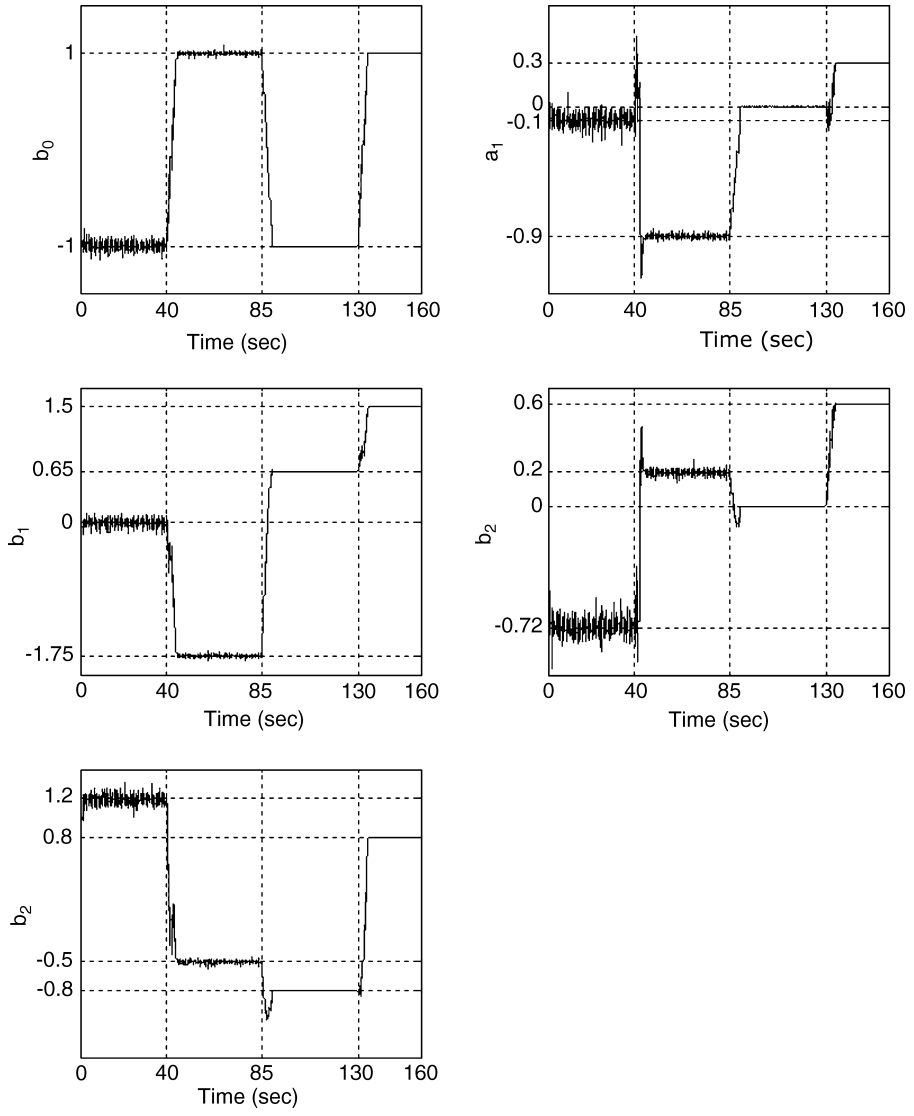


Fig. 3. Time evolution of the structural coefficients.

the ringing phenomenon, since it comes with the cross interaction between the unknown dynamics and the utilized identification method.

Lastly in this section, the computational burden of the algorithm is analyzed. Unlike LMS algorithm, the method presented does not need to store some history of

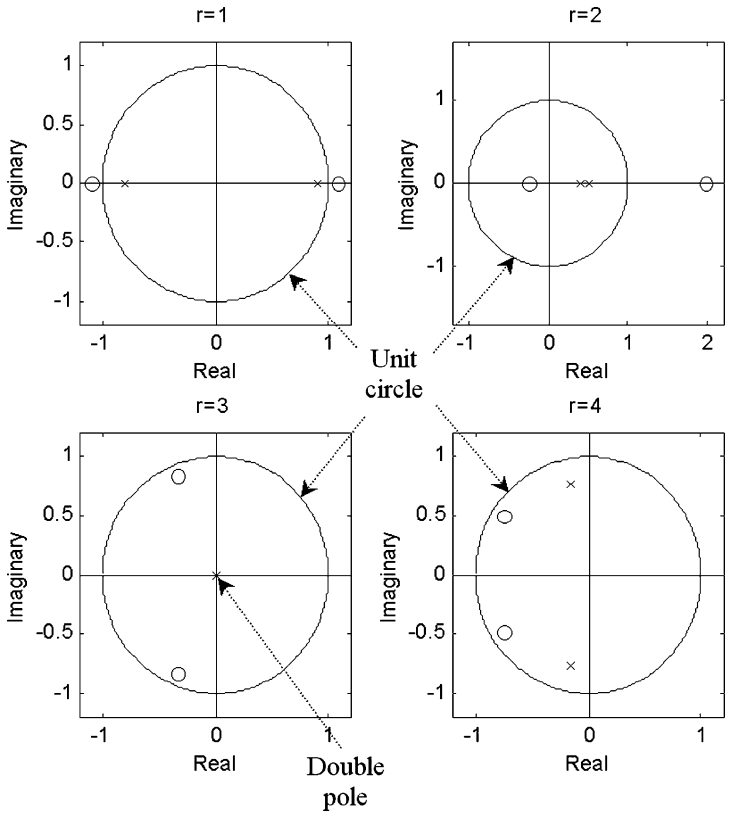


Fig. 4. Locations of the poles and the zeros of the structures activated during the successive intervals.

the system under investigation and operates on-line. Furthermore, no matrix inversion is needed throughout the operation. For the structure studied in this paper, i.e., $N = 2$ and $M = 2$, a single forward pass from (1) with tuning of parameters with (30) requires 46 floating point operations (flops), which is a quite promising value for real-time realizations when considered with the accurate identification performance. In Figure 5, the required number of flops has been illustrated for varying values of $M$ and $N$. Clearly, the computational cost is not excessive but the most important aspect of the proposed approach is the behavior of the increase in complexity as the dimensionality of the parameter vector increases. When the cost graph is sketched as a surface, it can be seen that the complexity linearly depends on the parameters $N$ and $M$. More precisely, we have found that the required number of flops is given by $8N + 8M + 14$, which is $O(M + N)$, i.e., the required number of flops will increase

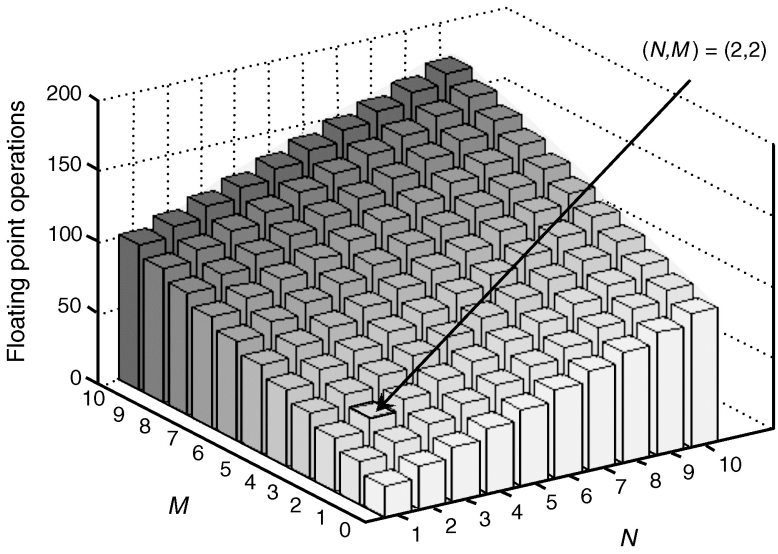


Fig. 5. Computational requirement of the algorithm for varying values of $M$ and $N$.

linearly as the dimension of the parameter vector increases, and this aspect constitutes a great advantage in dealing with higher order processes.

## 4. CONCLUSIONS

The lack of sufficient knowledge about a system of interest requires an in depth investigation procedure for developing mechanisms, that need the parameters characterizing the behavior of the system. For this purpose, various approaches are used to identify the system parameters. However, some of these schemes suffer from the computational complexity, while the applicability of some are subject to the availability of a priori knowledge. If the goal of handling the uncertainties with high performance and low cost is considered, the use of VSS theory becomes more comprehensible.

The results obtained through some simulations have clearly stipulated that the extraction of the unknown parameters can be achieved neither by storing excessive amount of data nor by occupying the CPU oppressively.

The radical changes in the ARMA structure have appropriately been detected by the discussed algorithm and the corresponding parameter set have precisely been determined. The finite volume parameter evolution and the stability claims of the

proposed technique are proved and the theoretical results have been confirmed by the example presented. Future work aims to implement the proposed technique for processes operating in real-time.

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