Multivariable nonlinear model reference control of cement mills

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This paper presents a method for model reference control of a cement milling circuit that has previously been studied several times. The approach presented is based on an experimentally justified model of a cement milling circuit. We derive the form of the control vector with the goal of driving the response of the system to that of a desired model in a noisy operating environment. The paper demonstrates the selection of the reference dynamics and the derivation of the control laws. The approach is based on the Lyapunov theory, and the results observed justify the tracking and stability claims of the paper.

Key words: industrial control; Lyapunov stability; model reference control; nonlinear control

1. Introduction

Ensuring the compliance of the ground product with the increasingly demanding cement quality standards has been a core issue in the cement industry. Several parameters, like the strength after a certain period of time, percentage sulphate (SO$_3$) content, percentage tricalcium aluminate (C$_3$A) content or fineness of the cement material determine to what extent the final product satisfies the desired specifications. Obtaining a consistent quality on the other hand depends heavily on the control and optimization approach utilized on-site. Without loss of generality, the design and implementation of control schemes in cement milling processes are typically involved with the selection of several feed rates as the control variables and aims to maintain a desired load on the mill. Some studies have therefore focused on the clarification of operational properties of the milling
process in general and mill drives in particular. Therefore, the dynamic representation of cement milling processes relate variables like mill load, product flow rate, tailing flow rate and some other system parameters in a nonlinear fashion; consequently the synthesis of an appropriate command and control scheme entails the expertise of nonlinear control systems.

Control of cement milling processes has been the focus of a number of research studies. The approaches postulated in the area of nonlinear control have extensively been applied, some of which have been exemplified in detail in the cited references. In particular, the model used in this paper has constituted a prime example due to the strong interdependencies between the variables involved. The model has three state variables and two control inputs, and despite its representational simplicity, the dynamics are quite complex and a good control performance can only be achieved if a suitable co-ordination between the two control inputs can be established and maintained.

The first results on this system have been presented by Van Breusegem et al. (1994, 1996), which are based on on-site experimentation of the system, and which constitute a basis for the dynamic model presented in Magni et al. (1999). It has been shown in Van Breusegem et al. (1994, 1996) that a linear quadratic control scheme based on the minimization of several system-specific performance criteria could lead to admissible results. In Magni et al. (1999), the multivariable predictive control of the system is studied. The problems associated with the plugging phenomenon and a robust control scheme is studied by Grognard et al. (2001). One prime conclusion reported in Grognard et al. (2001) is the necessity to include mill load in the state feedback information. This issue has further been discussed in Boulvin et al. (1999), which highlights the fact that choosing the mill load is not optional, instead it is a necessity. Dagci et al. (2001) have used the same model to test the control performance of a sliding mode-based controller design approach. It has been observed that an acceptable response could be obtained using the set-point values given in Magni et al. (1999). A recent work reporting the output feedback and tracking with model predictive control exemplifies the application of stabilizing receding horizon control concept on the mentioned cement milling circuit (Magni et al., 2001).

This paper is organized as follows: the second section describes briefly the dynamics of the cement milling circuit. The adopted reference model is introduced in the third section and the synthesis of the control signals is presented in the fourth section. In the fifth section, we formulate the error dynamics for all three states when the observations are noisy. The sixth section is devoted to the justification of the proposed scheme and the concluding remarks are given at the end of the paper.

2. Dynamics of the cement milling circuit

The dynamical model of the system is described by three coupled and nonlinear differential equations as given in (1)–(3). The states of the system are characterized by the mill load denoted by \( z \) (in tons), the product flow rate denoted by \( y_f \) (in tons/h) and the tailings flow rate denoted by \( y_r \) (in tons/h). On the other
hand, $\varphi$ is the output flow rate of the mill and $d$ denotes the relative hardness of the material inside the mill with respect to the nominal one, which is unity. The system has two control inputs, denoted by $u$ (tons/h), the feed flow rate and, $v$ (rpm), the classifier speed.

$$\dot{z} = -\varphi(z,d) + u + y_r$$  \hspace{1cm} (1)$$
$$T_f \dot{y}_f = -y_f + (1 - \alpha(z,v,d))\varphi(z,d)$$  \hspace{1cm} (2)$$
$$T_r \dot{y}_r = -y_r + \alpha(z,v,d)\varphi(z,d)$$  \hspace{1cm} (3)

where the functions $\varphi(z,d) = \max(0, -dK_wz^2 + K_wz)$ (tons/h) and

$$\alpha(z,v,d) = \frac{\varphi(z,d)^m v^n}{K_a + \varphi(z,d)^m v^n}$$  \hspace{1cm} (4)

In the above, $K_a = 570^m 170^n (570/450 - 1)$ (tons/h)$^m$ rpm$^n$, $m = 0.8$, $n = 4$, $K_{w1} = 0.1116$ (tons/h)$^{-1}$, $K_{w2} = 16.50$ h$^{-1}$, $T_f = 0.3$ h, $T_r = 0.01$ h and the nominal value of $d$ is unity. The variables $T_f$ and $T_r$ stand for the time constants for product flow rate and tailings flow rate dynamics, respectively. A schematic representation of the process is depicted in Figure 1, and a detailed description of the system dynamics and the results regarding the experimental justification of above terms can be found in Van Breusegem et al. (1996) and Magni et al. (1999).

Since the dynamics of the system is highly nonlinear and since the dynamical representation of the system is not affine in one of the control inputs, the design of a controller meeting the stability, tracking and robustness objectives is a challenge. This fact motivates control specialists to use the model as a test bed for novel control strategies.

The control problem is to enforce the system states to follow the states of a reference model by appropriately altering the two control inputs. At the first

![Figure 1 Schematic diagram of the cement milling circuit](image-url)
glance, it can be seen that the designer could choose two of the three state variables independently, as the behaviour of the third state variable would be determined upon the selection of other two. However, it is emphasized in Van Breusegem et al. (1996) and Boulvin et al. (1999) that the choice of $y_f$ and $y_r$ may lead to unachievable values for $\varphi$, and it is suggested in Boulvin et al. (1999) that keeping $y_f$ and $z$ under control is a necessity. In this paper, we adopt the same reasoning and proceed parallel to this idea.

3. Reference model

In choosing the reference dynamics for $z$ and $y_f$, we consider the following design requirements:

1) the reference model for each state, whose variables are represented with a subscript $m$, must be stable and must follow the command signal. Since the differential equations in (1)–(3) are of first order, we choose the corresponding reference dynamics as a first-order system;

2) the response imposed by the reference system must not be faster than what could be achieved by the actual system, i.e., the time constants must be compatible.

Denote the command signals for reference mill load ($z_m$) and the reference product flow rate ($y_{mf}$) states by $r$ and $f$, respectively. If for some $t_c > 0$, $z_m(t_c) = r(t_c)$ is satisfied, it is obvious that (5) is satisfied $\forall t \geq t_c$. Furthermore, the prescribed dynamics in (5) satisfies the two preliminary requirements stated above.

\[
\dot{z}_m = -z_m + r
\]  

Similar reasoning applies to the dynamics governing the product flow rate $y_{mf}$. The reference dynamics in (6) constitutes a good candidate satisfying the two requirements above.

\[
T_f \dot{y}_{mf} = -y_{mf} + f
\]  

As indicated in Van Breusegem et al. (1996) also, once two out of three state variables are kept under control, the behaviour of the third state is determined by the first two. Therefore, the same reasoning applies in designing the reference model. In what follows, we formulate the control signals and extract the imposed behaviour on the tailings flow rate ($y_{mr}$) component of the model, which is to be followed by the $y_r$ dynamics of the actual plant.

4. Synthesis of the control signal

Assume the third state variable ($y_{mr}$) of the model has the following dynamics, with $Q$ being a real-valued smooth function of the state variables, command signals and system parameters.

\[
T_r \dot{y}_{mr} = Q
\]
Consider the following Lyapunov function candidate:

\[ V = \frac{1}{2} e_z^2 + \frac{T_f}{2} e_{yf}^2 + \frac{T_r}{2} e_{yr}^2 \]  

where

\[ e_z = z - z_m \]  
\[ e_{yf} = y_f - y_{nf} \]  
\[ e_{yr} = y_r - y_{nr} \]

Evaluating the time derivative of the Lyapunov function in (8) yields the following:

\[ \dot{V} = e_z (\dot{z} - \dot{z}_m) + T_f e_{yf} (\dot{y}_f - \dot{y}_{nf}) + T_r e_{yr} (\dot{y}_r - \dot{y}_{nr}) \]  

Dropping the arguments of \( \alpha \) and \( \varphi \), and inserting (1)–(3) and (5)–(7) into (12) gives the quantity below:

\[ \dot{V} = e_z (-\varphi + u + y_r + z_m - r) + e_{yf} (-y_f + (1-\alpha) \varphi + y_{nf} - f) + e_{yr} (-y_r + \alpha \varphi - Q) \]  

One suitable formulation for \( u \) and \( v \) inputs can be performed by enforcing the following pair of equalities, which are chosen according to (13):

\[ -\varphi + u + y_r + z_m - r = -e_z \]  
\[ -y_f + (1-\alpha) \varphi + y_{nf} - f = -e_{yf} \]

If \( e_z \) and/or \( e_{yf} \) are nonzero, the two equalities in (14) ensure the negativity of the first two terms of the summation in (13). Substituting the two equalities in (14) into (13) leads to

\[ \dot{V} = -e_z^2 - e_{yf}^2 + e_{yr} (-y_r + \alpha \varphi - Q) \]  

and lets us formulate the control signals uniquely as

\[ u = \varphi - y_r - z + r \]  
\[ v = K_\alpha^{1/n} \left( \frac{\varphi}{f} - 1 \right)^{1/n} \varphi^{-m/n} \]

Note that with this pair of control signals, several internal relations are automatically created. Inserting (17) into (4) yields \( \alpha(z,v,d) = 1 - f/\varphi(z,d) \) in the controlled system. Consequently, the relation \( \alpha(z,v,d)\varphi(z,d) = \varphi(z,d) - f \) is automatically established in the closed loop. This has two practical consequences; the immediate one is \( v \in \mathbb{R} \ \forall t \) if \( \varphi(z,d) > 0 \). Secondly, if (3) is rewritten with this relation, one obtains the dynamics in (18) in the closed loop:

\[ T_r \dot{y}_r = -y_r + \varphi(z,d) - f \]  

In reality, the model corresponding to this behaviour must be

\[ T_r \dot{y}_{nr} = -y_{nr} + \varphi_{nL}(z_{nL,d}) - f \]
where \( \varphi_m(z_m,d) = \max(0, -dK_1z_m^2 + K_2z_m) \). The right-hand side of (18) clarifies that \( Q = -y_{mr} + \varphi_m(z_m,d) - f \) as seen in (7). Knowing \( \alpha(z,v,d)\varphi(z,d) = \varphi(z,d) - f \) and inserting this into (15) lets us proceed as follows:

\[
\dot{V} = -e_z^2 - e_yf + e_yr(-y_r + \alpha\varphi - (-y_{mr} + \varphi_m - f))
\]

\[
= -e_z^2 - e_yf - e_yr(\alpha\varphi - \varphi_m + f)
\]

\[
= -e_z^2 - e_y^2 - e_yr(\varphi - f - \varphi_m + f)
\]

\[
= -e_z^2 - e_y^2 - e_yr(\varphi - \varphi_m)
\]

(20)

Since \( T_f < 1 \) and \( T_r < 1 \), we can rewrite and rearrange (20) as follows:

\[
\dot{V} < -e_z^2 - T_f e_y^2 - T_r e_yr(\varphi - \varphi_m)
\]

\[
= -2V + \Xi
\]

where \( \Xi = e_yr(\varphi - \varphi_m) \). Having the Lyapunov system \( \dot{V} < -2V + \Xi \) in mind, one can write the below inequality that characterizes the time solution to this Lyapunov system:

\[
0 < V(t) < \exp(-2t)V(0) + \int_0^t \exp(-2(t - \sigma))\Xi(\sigma)d\sigma
\]

(22)

The selection of \( u \) given in (16) forces \( z \to z_{mr} \) hence \( \varphi \to \varphi_{mr} \). Using (18) and (19) with \( \alpha(z,v,d)\varphi(z,d) = \varphi(z,d) - f \), one can write \( T_r\dot{e}_{yr} = -e_{yr} + (\varphi - \varphi_m) \). Clearly, \( \varphi \to \varphi_m \) ensures \( e_{yr} \to 0 \), and hence \( \Xi(t) \to 0 \). Furthermore, with the selected control inputs, the states \( y_f \) and \( z \) are decoupled and are enforced to follow the corresponding model states. When the solution in (22) is considered with these conclusions, it becomes apparent that \( V(t) \) will follow \( \Xi(t) \) in the steady state, and \( \Xi(t) \) is enforced to converge to zero due to the design presented. Therefore, the inequality in (22) states that the time solution of the Lyapunov system \( \dot{V} < -2V + \Xi \) is globally uniformly ultimately bounded.

**Remark 1.** It must be noted that the designer can specify only two of the three state variables independently, and by appropriately designing the two control signals, the chosen two states can be maintained at desired levels. The time evolution in the third state is determined by the other two, which are explicitly under control. The same reasoning applies to the study of reference model determination. We have chosen two states of the reference model independently, and described their behaviour through two dynamical equations [refer to (5) and (6)]. This has let us choose the control signals. The choice of \( u \) and \( v \) has automatically determined the dynamics of the third state. Finally, the issue of stability has been studied to demonstrate that the system – with its imposed third state – is stable, and the time solution of the Lyapunov function is globally uniformly ultimately bounded.

**Remark 2.** The time solution of the Lyapunov function is enforced to follow an intermediate variable (\( \Xi \)) that converges to zero. This lets us conclude with the fact that the closed-loop system will follow the response of the reference model to the extent determined by the value of \( \Xi \) during the course of a control trial.

**Remark 3.** The proposed controller drives the behaviour of the plant to that of a
reference model. The technique is well known as model reference control (MRC). An extension of MRC exploits the strength of parameter adaptation. The interested reader is referred to Aström and Wittenmark (1993).

5. Effect of noise-corrupted observations

In this section, we analyse the consequences of the noisy observations. For this purpose, assume that the states of the system are corrupted by noise sequences additively, and denote these disturbance signals by $\zeta_z$, $\zeta_{zf}$ and $\zeta_{yr}$. The control signals in (16) and (17) will have the following forms:

$$u = \varphi(z + \zeta_z d + \zeta_d) - (y_r + \zeta_{yr}) - (z + \zeta_z) + r$$

(23)

$$v = K_{1/n}(\frac{\varphi(z + \zeta_z d + \zeta_d)}{f} - 1)^{1/n} \varphi(z + \zeta_z)^{-m/n}$$

(24)

where $\zeta_d$ is the noise on measured relative material hardness parameter $d$. Application of the above controls to the system of (1)–(3), and denoting

$$w_z = \max(0, -(d + \zeta_d)K_{\varphi}(z + \zeta_z)^2 + K_{\varphi^2}(z + \zeta_z))$$

the dynamics of the model following errors given in (9)–(11) are obtained as follows:

$$\dot{e}_z = -e_z + e_1$$

(25)

$$T_{ze} = -e_{zf} + e_2$$

(26)

$$T_{ze} = -e_{yr} + e_3$$

(27)

where

$$e_1 = -(\zeta_{yr} + \zeta_z) + (\varphi - \varphi)$$

(28)

$$e_2 = \frac{\varphi}{1 + \left(\frac{\varphi}{\varphi}\right)^m \left(\frac{\varphi}{f} - 1\right)} - f$$

(29)

$$e_3 = \frac{\varphi}{1 + \left(\frac{\varphi}{\varphi}\right)^m \left(\frac{\varphi}{f} - 1\right)} - \varphi + f$$

(30)

Clearly the expressions in (25)–(27) suggest that the error dynamics are stable in each component. Having this in front of us, the way in which the proposed scheme responds to corrupted observations can be examined through analysing the behaviours of the quantities $e_1$, $e_2$ and $e_3$, which are explicitly given in (28)–(30). Clearly, if $\|\zeta_z \zeta_{zf} \zeta_{yr}\| \to 0$, then $\varphi \to \varphi$ and this leads to $e_1 \to 0$, $e_2 \to 0$ and $e_3 \to \varphi - \varphi_m$. Since $\varphi \to \varphi_m$ is enforced by the presented design, all error components tend to be in the vicinity of zero as time progresses. In the cases where the norm $\|\zeta_z \zeta_{zf} \zeta_{yr}\|$ is not negligibly small, the tracking errors will exhibit behaviours characterized by (25)–(27).
Knowing the behaviour of above norm value may have substantial importance for on-site and real-time experimentation, which absolutely necessitates the knowledge of to what extent the task is accomplished. From this point of view, (28)–(30) provides a useful guidance about what sensor precision is needed and/or what estimation/filtering tools are needed to implement the controller discussed in this paper.

6. Justification of the proposed scheme – simulations

During the simulations, the plant is kept under an ordinary feedback loop, and the control signals are generated using the noisy observations of the system response. The disturbance corrupting the $z$ state of the system has variance equal to 0.0029 and that corrupting the state $y_r$ has variance equal to 0.0028. The maximum amplitude of the noise sequences is equal to 0.2 and both sequences have zero means and are Gaussian distributed. The relative material hardness parameter has been chosen as $d(t) = 1 + 0.34 \sin(2\pi t/20)$ and the associated measurement noise has been set as $z_d(t) = 0.005 \sin(20\pi t/90)$. Clearly, the given disturbance scenario stipulates that the state variables and the hardness parameter required to construct the control signals are severely corrupted. We set the simulation step size to 0.005 h, which is reasonably smaller than the time constants in (1)–(3), and simulated the control system for 90 h with a fourth-order Runge–Kutta solver.

Initially $y_f(0) = 140$, $y_r(0) = 0$ and $z(0) = 50$; on the other hand, the reference model states have initially been set as $y_{mf}(0) = 120$, $y_{mr}(0) = 20$ and $z_m(0) = 40$. These values have been selected according to the typical values that appear in the cited references. In the upper subplot of Figure 2, the command signal ($r$), the

![Figure 2](image.png)

**Figure 2** The command signal ($r$), the response of the reference model and the observed mill load behaviour
response of the reference model ($z_m$) and the response of the system are illustrated together. After approximately 5 h, the value of the plant output comes admissibly close to that of the model output. In order to clarify the tracking claim of the paper, the lower subplot depicts the model following error in state $z$. The variation in the plant output is due to the uncertainty on the relative material hardness parameter. Our work has shown that for some time interval, if the $\sup |\zeta_d|$ increases, the magnitude of the deviations increases.

Similarly for the product flow rate ($y_f$), in the top row of Figure 3, the command signal ($f$), the response of the reference model ($y_{mf}$) and the response of the system are illustrated. The model-following error is shown in the bottom row of the figure. It is apparent that the error quickly converges near to zero and remains thereafter. Similarly, the fluctuations on the system response are primarily due to the uncertainty on $d$.

The results regarding the third state ($y_r$) are visualized in Figure 4. The bottom subplot of the figure demonstrates that the model-following error comes very close to zero after a fast transient effective during the initial phase. A tiny adverse effect of nonzero $\zeta_d$ is also apparent from this figure.

The applied control signals are depicted in Figure 5, which suggests that the scheme is reasonably safe in terms of the control signal magnitudes. Although there seem mildly fast changes in the control signal, the reader must notice that the horizontal axis is in hours, and the state variables are corrupted. Therefore, observing such a behaviour in the scale shown is parallel to what is imposed by the operating conditions.

Aside from what are presented in Figures 2-5, we have repeated the simulations with perfect knowledge of $d$ without changing the state disturbances. Unsurpris-
Figure 4  The response of the reference model \( y_{mr} \) and that of the milling circuit

Figure 5  Applied control signals, \( u \) at the upper subplot and \( v \) at the lower subplot
ingly, the results obtained in such an idealized case are highly promising until we increase the magnitude of fluctuations in \( d \) to 0.75. The state variables become almost indistinguishable from their counterparts observed from the reference model. This stipulates that the material hardness parameter is one of the key parameters determining the performance of cement production lines and the uncertainty, which significantly influences the quality of the end product.

According to our observations, the most important reason that this problem is challenging are the variations in the material hardness parameter, whose behaviour is illustrated in Figure 6(a). We adopt a periodically changing relative material hardness, the value of which ranges from unity to 0.655 to 1.345. The controller uses the corrupted value of it, \( d + \zeta \), shown in Figure 6(a). Referring to the discussion in section 3, we have claimed that the constructed control signals would enforce \( z \rightarrow z_m \), and hence \( \phi(z,d) \rightarrow \phi_m(z_m, d) \). In Figure 6(b), we illustrate the behaviour of \( \phi_m/\phi \), which confirms the above-quoted convergence statement. Clearly the tendency of this quantity is to lie around unity.

A rather implicit effect of \( d \) and the operating region can be seen on Figure 7. We plot the behaviour of \( \phi(z,d) \) for \( d = 0.66 \) and \( d = 1.34 \), which are the two bounds for \( d \) studied in this paper. With this in mind, in order to ensure \( \nu \in \Re \) we need \( \phi(z,d) > 0 \). It is clearly visible that this restricts the operating region in terms of \( z_m \) dynamics. Since \( z \) is forced to follow \( z_m \), the design must take this into account such that a positive output flow rate is maintained by properly selecting the command signal \( r \). In the presented simulation results, it is seen that the \( z \) versus \( \phi(z,d) \) pair, and \( z_m \) versus \( \phi_m(z_m,d) \) pair move in the ABCD region of Figure 7. Since the chosen reference model is a first-order one, there will be no overshoot and the margins shown on Figure 7 will lead to a safe operation ensuring \( \phi(z,d) > 362 > 0 \).

A final issue in this section should be a brief comparison of the results with those appearing in the relevant literature. Our first comparison is the simplicity of the design, which clearly emphasizes that a reference model can be devised so

![Figure 6](image)

**Figure 6** (a) The variation in the hardness of the material in the mill \( (d + \zeta) \); (b) behaviour of the quantity \( \phi_m/\phi \), which is maintained around unity
that the two basic requirements are met. In terms of this, the method we propose is the simplest available in the cited literature. The system states then keep following the reference model states, thereby satisfying the stability requirements. In terms of tracking ability, the method is very good; however, the performance is dependent upon the certainty on the material hardness. This means that the control system with the presented technique is not as robust against uncertainty on material hardness as with those presented previously, e.g., Grognard et al. (2001). When the application is confined to the given model, the presented technique results in better tracking precision and transient response than those studied in Magni et al. (1999) and Van Breusegem et al. (1994).

7. Conclusions

A nonlinear control strategy for a cement milling process is studied in this paper. The constructed forms of the control signals are reasonably simple and have resulted in good performance in terms of the model-following capability.

Considerable amounts of observation noise, large initial errors and response with time-varying system parameters are studied, and it has been observed that the suggested strategy results in good performance in terms of all these control-specific measures.

The results have shown that the relative material hardness parameter is a key design variable, the certainty on the knowledge of which is determining the performance dominantly. Future research on this issue aims to devise control schemes minimizing the sensitivity to the relative material hardness parameter.
References


