

# Discrete Time Fuzzy Sliding Mode Control of a Biochemical Process

MEHMET ÖNDER EFE

Department of Electrical and Electronics Engineering  
 TOBB Economics and Technology University  
 Söğütözü Ankara, TR06560  
 TURKEY  
 onderefe@etu.edu.tr

**Abstract:** This paper presents a novel approach for designing discrete time fuzzy sliding mode controllers for a class of nonlinear systems. The motivation of the study is to explain how a sliding regime can be created in the behavior of a computer-controlled SISO nonlinear system, the details of the dynamic representations of which is unknown. The scheme exploits the regional information contained in the fuzzy quantification of the error vector. The bottom-up approach is proven to be computationally efficient for real time control applications requiring the handling of uncertainty and impreciseness and yielding a good tracking performance under the presence of time-varying plant parameters and observation noise. The analytical claims have been confirmed on a biochemical process model. The results obtained through simulations justify the theoretical claims.

**Key-words:** Discrete Time Sliding Mode Control, Fuzzy Control, Parameter Tuning, Biochemical Process Control

## 1 Introduction

Variable structure systems with a sliding mode have extensively been studied by many researchers. The invariance properties and insensitivity to parameter variations are the most prominent features that are observed from a variable structure control system. The method is based on the construction of a two-sided decision mechanism, the boundary of which is described by a function called switching function. Particularly for continuous time systems, the design procedure is well established and many aspects of the control technique have already been analyzed in detail [1-4]. The design and implementation of variable structure control systems in discrete time have later been considered, and still the framework of Discrete Time Sliding Mode Control (DTSMC) is in progress towards figuring out the design issues under a set of conditions describing the problem. One of the notable works discussing the stability issues in DTSMC is presented in [5], in which the sufficient conditions for convergence are discussed.

This paper proposes a fuzzy control scheme for biochemical systems. The method is based on the tuning of the defuzzifier parameters while maintaining the membership functions time-invariant. The plant is under an ordinary feedback loop and the controller processes the plant state at discrete instants of time. The contribution of the paper is the postulation of a novel error extraction scheme and the use of a particular tuning mechanism to achieve a set of performance specifications under the presence of noise and uncertainties. In the second section, a thorough analysis of the control system is presented. At the outset, the class to which the plant belongs is described, the

structure of the fuzzy controller is presented and the adaptation strategy is discussed. In the third section, the bioreactor control problem is described. The conclusions are presented at the end of the paper.

## 2 Analysis of the Control System

Consider the control system structure depicted in Fig. 1.

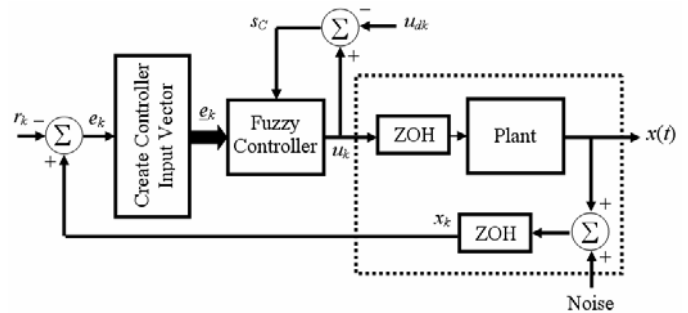


Fig. 1. Structure of the Control System

The behavior of the plant under control has the structure given by (1), in which a subscript  $k$  stands for time index.

$$x_{k+1} = f(x_k, \dots, x_{k-n}) + g(x_k, \dots, x_{k-c})u_k \quad (1)$$

where  $n$  and  $c$  are some integers determining the effect of delay depth, furthermore, the function  $f$  is unknown while  $g$  is available. The system above is a SISO one and can more compactly be written as  $x_{k+1} = f_k + g_k u_k$ . According to Fig. 1, the error on the output  $x$  at time  $k$  is

expressed as  $e_k = x_k - r_k$ , where  $r(t)$  is the command signal. Define the switching function as

$$s_k = \alpha_0 e_k + \alpha_1 e_{k-1} + \dots + \alpha_p e_{k-p} \quad (2)$$

where the dynamics determined by  $s_k=0$  is stable, i.e. the roots of the polynomial  $\alpha_0 z^p + \alpha_1 z^{p-1} + \dots + \alpha_p$  lie inside the unit circle. Now adopt a closed loop switching dynamics described generically as  $s_{k+1} = Q(s_k)$ , and evaluate  $s_{k+1}$ , which is given below.

$$\begin{aligned} s_{k+1} &= \alpha_0 e_{k+1} + \alpha_1 e_k + \dots + \alpha_p e_{k-p+1} \\ &= \alpha_0 (f_k + g_k u_k - r_{k+1}) + \alpha_1 e_k + \dots + \alpha_p e_{k-p+1} \end{aligned} \quad (3)$$

Using  $s_{k+1} = Q(s_k)$  and solving for  $u_k$  gives the control sequence formulated as below.

$$u_k = g_k^{-1} (r_{k+1} - f_k + \alpha_0^{-1} (Q(s_k) - (\alpha_1 e_k + \dots + \alpha_p e_{k-p+1}))) \quad (4)$$

If the values of the functions  $f_k$  and  $g_k$  were available, the application of this sequence to the system of (1) would result in  $s_{k+1} = Q(s_k)$ , where  $Q$  must satisfy the condition below to ensure reaching [5].

$$s_k (s_{k+1} - s_k) = s_k (Q(s_k) - s_k) < 0 \quad (5)$$

If the condition above is satisfied for  $\forall k \geq 0$ , the system is driven towards the dynamics characterized by  $s_k=0$ . However in practice,  $s_k=0$  is rarely observed as the problem is described in discrete-time. A realistic observation is  $|s_k| < \varepsilon$ , where  $\varepsilon$  is some positive number. In the literature, this phenomenon is called quasi-sliding mode, or equivalently pseudo-sliding mode. This mode has useful invariance properties in the face of uncertainties and time variations in the plant and/or environment parameters. Once the quasi-sliding regime starts, the error signal behaves as what is prescribed by  $|s_k| < \varepsilon$ .

### 2.1 Computing the Control Error

Consider Fig. 1, which demonstrates that the quantity  $s_{Ck}$  would be the error on the applied control signal if we had a supervisor providing the desired value of the control denoted by  $u_{dk}$ . However, the nature of the problem does not allow the existence of such supervisory information, instead of it, the designer is enforced to extract the value of  $s_{Ck}$  from the available quantities. In what follows, we present a method to extract the error on the control signal.

**Assumption 2.1:** The functions  $f_k$  and  $g_k$  of (1) are such that a desired quasi-sliding mode can be created with a suitable selection of the design parameters, more

explicitly, we assume that the DTSMC task is achievable.

**Remark 2.2:** A control sequence leading to DTSMC can be formulated if the dynamics of the system in (1) is totally known or if the nominal system is known with the bounds of the uncertainties. It must be noted that the disturbances and uncertainties are assumed to enter the system through the control channel [1]. When the control sequence in (4) is applied to the system of (1), we call the resulting behavior as the *target DTSMC* and the input signal leading to it as the *target control sequence* ( $u_k$ ). If at least the explicit forms of the nominal representations of  $f_k$  and  $g_k$  are not known, it should be obvious that the target control sequence cannot be constructed by following the traditional DTSMC design approaches.

**Definition 2.3:** Given an uncertain plant, which has the structure described as in (1), and a command trajectory  $r_k$  for  $k \geq 0$ , the input sequence denoted by  $u_{dk}$  satisfying the following difference equation is defined to be the *idealized control sequence*, and the difference equation itself is defined to be the *reference DTSMC model*.

$$\begin{aligned} r_{k+1} &= f_{dk}(r_k, \dots, r_{k-n}, u_{d,k-1}, \dots, u_{d,k-b}) + \\ &g_k(r_k, \dots, r_{k-c}, u_{d,k-1}, \dots, u_{d,k-d}) u_{dk} \end{aligned} \quad (6)$$

or more compactly  $r_{k+1} = f_{dk} + g_k u_{dk}$ . Mathematically, the existence of such a model and the sequence means that the system of (1) perfectly follows the command trajectory ( $r_k$ ) if both the idealized control sequence ( $u_{dk}$ ) is known and the initial conditions are set as  $x_0=r_0$ , more explicitly  $e_k \equiv 0$  for  $\forall k \geq 0$ . Undoubtedly, the reference DTSMC model is an abstraction as the functions appearing in it are not available, however, the concept of idealized control sequence should be viewed as the synthesis of the command signal  $r_k$  from the time solution of the difference equation in (6).

**Fact 2.4:** If the target control sequence formulated in (4) were applied to the system of (1), the idealized control sequence would be the steady state solution of the control signal, i.e.  $\lim_{k \rightarrow \infty} u_k = u_{dk}$ . However, under the assumption of the achievability of the DTSMC task, the difficulty here is again the unavailability of the functional forms of the functions  $f_k$  and  $g_k$ . Therefore, the aim in this subsection is to discover an equivalent form of the discrepancy between the control applied to the system and its target value by utilizing the idealized control viewpoint. This discrepancy measure is denoted by  $s_{Ck} = u_k - u_{dk}$ . If the target control sequence of (4) is rewritten by using (6), one gets (7) and setting  $\Delta f_k = f_k - f_{dk}$  gives (8)

$$\begin{aligned} u_k &= g_k^{-1} (f_{dk} + g_k u_{dk} - f_k + \\ &\alpha_0^{-1} (Q(s_k) - (\alpha_1 e_k + \dots + \alpha_p e_{k-p+1}))) \end{aligned} \quad (7)$$

$$u_k = g_k^{-1} \Delta f_k + u_{dk} + (\alpha_0 g_k)^{-1} (Q(s_k) - (\alpha_1 e_k + \dots + \alpha_p e_{k-p+1})) \quad (8)$$

The target control sequence becomes identical to the idealized control sequence, i.e.  $u_k \equiv u_{dk}$  as long as  $\alpha_0 \Delta f_k - Q(s_k) + \alpha_1 e_k + \dots + \alpha_p e_{k-p+1} = 0$  holds true for  $\forall k \geq 0$ . However, this condition is of no practical importance as we do not have the analytic forms of the functions  $f_k$  and  $g_k$ . Therefore, one should consider this equality as an equality to be enforced instead of an equality that holds true all the time, because its implication is  $s_{Ck} = 0$ , which is the ultimate goal of the design. It is obvious that to enforce this equality to hold true will let us synthesize the target control sequence, which will eventually converge to the idealized control sequence by the adaptation algorithm yet to be discussed. Consider  $s_{k+1}$  given below.

$$\begin{aligned} s_{k+1} &= \alpha_0 g_k s_{Ck} + \alpha_0 \Delta f_k + \alpha_1 e_k + \dots + \alpha_p e_{k-p+1} \\ &= \alpha_0 g_k s_{Ck} + Q(s_k) \end{aligned} \quad (9)$$

Solving the above equation for  $s_{Ck}$  yields the following

$$s_{Ck} = (\alpha_0 g_k)^{-1} (s_{k+1} - Q(s_k)) = u_k - u_{dk} \quad (10)$$

The interpretation of the above control error measure is as follows: Since we are in pursuit of enforcing  $s_{k+1} = Q(s_k)$  in the closed loop, during the time until which this equality does not hold true, the applied control sequence is not the sought one. However, if an adaptation strategy enforces (10) to approach zero, this enforces  $s_{k+1} \rightarrow Q(s_k)$ , consequently  $u_k \rightarrow u_{dk}$  as  $k$  increases.

**Remark 2.5:** The reader must here notice that the application of  $u_{dk}$  for  $\forall k \geq 0$  to the system of (1) with zero initial errors will lead to  $e_k \equiv 0$  for  $\forall k \geq 0$ , on the other hand, the application of  $u_k$  for  $\forall k \geq 0$  to the system of (1) will lead to  $s_k = 0$  for  $\forall k \geq k_h$ , where  $k_h$  is the hitting time index, at which the quasi-sliding regime starts. Therefore, the adoption of (10) as the equivalent measure of the control error loosens  $e_k \equiv 0$  for  $\forall k \geq 0$  requirement and enforces  $s_{k+1} \rightarrow Q(s_k)$ . Consequently, the tendency of the control scheme will be to generate the target DTSMC sequence of (4).

**Remark 2.6:** Referring to (10), it should be obvious that if  $s_{Ck} (s_{Ck+1} - s_{Ck}) < 0$  is satisfied  $s_k (s_{k+1} - s_k) < 0$  is enforced. Verbally, if the control signal approaches the target control sequence, the DTSMC task is achieved and the plant follows the command signal.

## 2.2 Fuzzy Controller

Consider a two-input, one output fuzzy controller having  $R$  rules in the rule base, triangular membership functions

and product inference engine. The input-output relation of such a system is given by

$$u_k = \frac{\sum_{i=1}^R \beta_{ik} \prod_{j=1}^2 \mu_{ij}(e_{jk})}{\sum_{i=1}^R \prod_{j=1}^2 \mu_{ij}(e_{jk})} \quad (11)$$

where,  $e_{jk}$  is the  $j^{\text{th}}$  component of the input vector at time  $k$ ,  $\mu_{ij}$  is the  $i^{\text{th}}$  rule's  $j^{\text{th}}$  membership function and  $\beta_{ik}$  is the scalar conclusion of  $i^{\text{th}}$  rule and is adjustable. The fuzzy system above can more compactly be expressed as  $u_k = \underline{\beta}_k^T \underline{\Omega}_k$ , where  $\underline{\beta}$  and  $\underline{\Omega}$  are  $R \times 1$  vectors and  $\underline{\Omega}_k = \underline{w}_k (\sum_{i=1}^R \underline{w}_{ik})^{-1}$  with  $\underline{w}_{ik} = \prod_{j=1}^2 \mu_{ij}(e_{jk})$ . Set  $R=9$  and consider the fuzzy quantization depicted in Fig. 2. We set the membership functions time-invariant, and consider solely the adjustment of  $\underline{\beta}$  vector. According to Fig. 2,  $\sum_{i=1}^R \underline{w}_{ik} = 1$ , therefore  $\underline{\Omega}_k = \underline{w}_k$ .

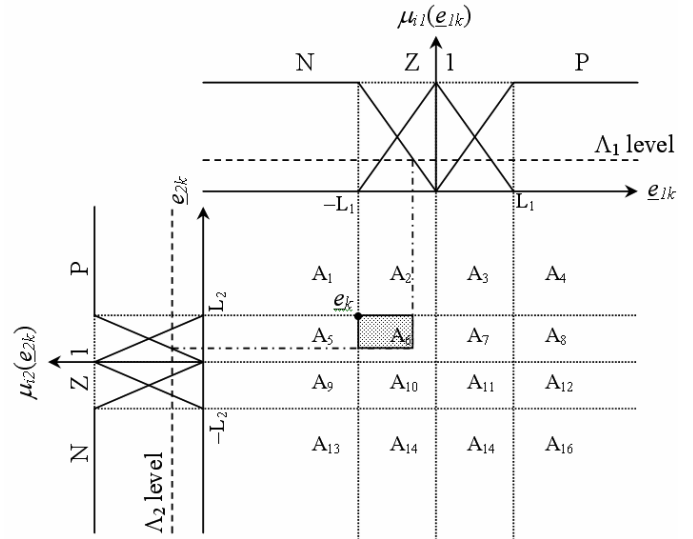


Fig. 2. Construction of the Membership Functions

## 2.3 Adaptation Mechanism

Define the following quantities:  $\underline{\Omega}_{k+1} = \underline{\Omega}_k + \underline{\Delta \Omega}_{k+1}$  and  $u_{dk+1} = u_{dk} + \Delta u_{dk+1}$ . In order not to violate the requirements of the physical reality, we impose  $\|\underline{\Delta \Omega}_k\| \leq 2B_{\Omega}$  for  $\forall k \geq 0$  and  $|\Delta u_{dk}| \leq 2B_{ud}$  for  $\forall k \geq 0$ , where  $B_{\Omega}$  and  $B_{ud}$  are some positive constants satisfying  $\|\underline{\Omega}_k\| \leq B_{\Omega}$  and  $|u_{dk}| \leq 2B_{ud}$  for  $\forall k \geq 0$  respectively. Furthermore, the adjustable parameter vector of the fuzzy controller is assumed to evolve bounded, i.e.  $\|\underline{\beta}_k\| \leq B_{\beta}$ , where  $B_{\beta}$  is some positive constant. Using these quantities, we set  $\zeta = B_{\beta} + 2B_{\Omega} + 2B_{ud}$  and assume that  $(\underline{\Omega}_k^T \underline{\Omega}_{k+1}) / (\underline{\Omega}_k^T \underline{\Omega}_k) > \Gamma > 0$  is satisfied for some subspace of the space  $\mathfrak{R}^2$ . The existence of such a  $\Gamma$  and the meaning of the assumption will be discussed later.

**Theorem 2.7:** For a discrete time SISO system of structure (1), the use of a 2-input one output fuzzy controller described in (11) with a parameter adaptation rule as described in (12) leads to  $s_{Ck}(s_{Ck+1} - s_{Ck}) < 0$ .

$$\underline{\beta}_{k+1} = \underline{\beta}_k - \gamma \frac{\underline{\Omega}_k}{\underline{\Omega}_k^T \underline{\Omega}_k} \text{sgn}(s_{Ck}) \quad (12)$$

where,  $\gamma$  is a positive constant satisfying  $\gamma > \zeta/\Gamma$ .

**Proof:**

$$\begin{aligned} s_{Ck}(s_{Ck+1} - s_{Ck}) &= s_{Ck} \left( \underline{\beta}_{k+1}^T \underline{\Omega}_{k+1} - u_{dk+1} - s_{Ck} \right) \\ &= s_{Ck} \left( \underline{\beta}_k^T \underline{\Omega}_k - u_{dk} + u_{dk} + \underline{\beta}_k^T \underline{\Delta \Omega}_{k+1} \right. \\ &\quad \left. - \gamma \frac{\underline{\Omega}_k^T \underline{\Omega}_{k+1}}{\underline{\Omega}_k^T \underline{\Omega}_k} \text{sgn}(s_{Ck}) - u_{dk+1} - s_{Ck} \right) \\ &= s_{Ck} \left( \underline{\beta}_k^T \underline{\Delta \Omega}_{k+1} - \gamma \frac{\underline{\Omega}_k^T \underline{\Omega}_{k+1}}{\underline{\Omega}_k^T \underline{\Omega}_k} \text{sgn}(s_{Ck}) - \Delta u_{dk+1} \right) \\ &\leq -\gamma \frac{\underline{\Omega}_k^T \underline{\Omega}_{k+1}}{\underline{\Omega}_k^T \underline{\Omega}_k} |s_{Ck}| + (B_\beta + 2B_\Omega + 2B_{u_d}) |s_{Ck}| \\ &= -\left( \gamma \frac{\underline{\Omega}_k^T \underline{\Omega}_{k+1}}{\underline{\Omega}_k^T \underline{\Omega}_k} - \zeta \right) |s_{Ck}| \\ &< -(\gamma\Gamma - \zeta) |s_{Ck}| < 0 \end{aligned} \quad (13)$$

This implies that the adaptation mechanism of (12) enforces the fuzzy controller to synthesize the target control sequence, which leads to the achievement of the prescribed DTSMC task, and the theorem is proved.

**Theorem 2.8:** There exists a strictly positive  $\Gamma$  if the motion in the 2-dimensional controller input space satisfies the conditions in (14) and (15).

$$|\mu_{i1}(\underline{e}_{1k+1}) - \mu_{i1}(\underline{e}_{1k})| \leq 1 - \Lambda_1 \quad (14)$$

$$|\mu_{i2}(\underline{e}_{2k+1}) - \mu_{i2}(\underline{e}_{2k})| \leq 1 - \Lambda_2 \quad (15)$$

where,  $0 < \Lambda_1 < 1$  and  $0 < \Lambda_2 < 1$ .

**Proof:** Since  $\gamma > \zeta/\Gamma$  and we assumed that  $(\underline{\Omega}_k^T \underline{\Omega}_{k+1})/(\underline{\Omega}_k^T \underline{\Omega}_k) > \Gamma > 0$ , we need is to evaluate the least value of  $(\underline{\Omega}_k^T \underline{\Omega}_{k+1})/(\underline{\Omega}_k^T \underline{\Omega}_k) > \Gamma$  and to show that it is strictly positive. Before going into the details, one has to notice from (14)-(15) that a binary change in any of the membership functions is prohibited. For example, if  $\mu_{i1}(\underline{e}_{1k})=1$  for some  $k$ , the value of  $\mu_{i1}(\underline{e}_{1k+1})$  can decrease at most to the level  $1-\Lambda_1$ . Referring to Fig. 2, let the input vector is to perform a transition from region  $A_1$  at time  $k$  to region  $A_6$ , at time  $k+1$ , and denote this transition by

$A_1 \rightarrow A_6$ . Clearly, the conditions in (14)-(15) require that the point  $\underline{e}_k$  in Fig. 2 can reach points in the shaded area at time  $k+1$ . This area is the largest area that can be reached from the region  $A_1$ , as the point  $\underline{e}_k$  is at the mutual neighborhood of the regions  $A_1$  and  $A_6$ . Having this in mind, we can claim that the least value of  $\underline{\Omega}_k^T \underline{\Omega}_{k+1}$  that can be observed from  $A_1 \rightarrow A_6$  transition is  $\Lambda_1 \Lambda_2$ . In obtaining this, one should note that it is sufficient to check the least value of  $\underline{\Omega}_k^T \underline{\Omega}_{k+1}$  because the supremum value of  $\underline{\Omega}_k^T \underline{\Omega}_{k+1}$  is unity. Once the minimal least value of  $\underline{\Omega}_k^T \underline{\Omega}_{k+1}$  for all possible transitions is constructed, a candidate  $\Gamma$  value can be set if the globally minimum value of  $\underline{\Omega}_k^T \underline{\Omega}_{k+1}$  is strictly positive. After straightforward manipulations, one can show that

$$\frac{\underline{\Omega}_k^T \underline{\Omega}_{k+1}}{\underline{\Omega}_k^T \underline{\Omega}_k} > \Lambda_1 \Lambda_2 \min \left( \left( 1 - \frac{\Lambda_1}{2} \right) \left( 1 - \frac{\Lambda_2}{2} \right), \Lambda_1 \Lambda_2 \right) \quad (16)$$

The result above ensures that a  $\Gamma > 0$  exists and it satisfies the inequality in (17).

$$0 < \Gamma < \Lambda_1 \Lambda_2 \min \left( \left( 1 - \frac{\Lambda_1}{2} \right) \left( 1 - \frac{\Lambda_2}{2} \right), \Lambda_1 \Lambda_2 \right) \quad (17)$$

The result we obtained proves Theorem 2.8 and confirms the stability claim of Theorem 2.7.  $\square$

**Remark 2.9:** A system of structure (1) in the feedback loop illustrated in Fig. 1 can be driven towards a quasi-sliding mode if the adopted fuzzy controller has the structure discussed in subsection 2.2 and if the adaptation mechanism is as given in (12). The proposed scheme extracts the error measure by using (10), which is to be used in the adjustment of the fuzzy controller parameters.

## 2.4 Practical Issues

### 2.4.1. Sampling Time

The multivariable functions  $f$  and  $g$  seen in the dynamics of the system of (1), must be smooth enough, and the command signal must be smooth enough in order not to cause jumps violating (14) and (15). This requirement is tightly dependent upon the sampling period  $T_s$ .

### 2.4.2. Causality

In (10), we have found that the error on the applied control at time  $k$  is  $s_{Ck} = (\alpha_0 g_k)^{-1} (s_{k+1} - Q(s_k))$ . However, the right hand side requires the value  $s_{k+1}$ . In the studied process control application, we set  $s_{Ck} = (\alpha_0 g_{k-1})^{-1} (s_k - Q(s_{k-1}))$ , the right hand side of

which is actually the control error at time  $k-1$ . Assuming this form as practically equivalent measure of the control error, we introduce some amount of uncertainty into the control system, which can be represented in the system dynamics that has already been assumed to be unknown.

**2.4.3. Actuation Speed**

Another important issue is the actuation speed of the system under control, i.e. the ability to respond to what is imposed timely. Since we assume that the details concerning the dynamic model of the system are unavailable, what causes a difficulty from a practical point of view is the selection of  $Q(s_k)$ , which characterizes the behavior during the reaching mode. The parameters of this quantity can only be set by trial-and-error due to the lack of system-specific details.

In the application example, we utilize  $s_{k+1} = (1 - \lambda_1 T_s) s_k - \lambda_2 T_s \text{sgn}(s_k)$ , where  $\lambda_1 > 0$ ,  $\lambda_2 > 0$  and  $(1 - \lambda_1 T_s) > 0$ . The sign function is smoothed out for eliminating the adverse effects of chattering phenomenon.

**3 Application Example**

The model used in this subsection is a bioreactor defined by Ungar [6] as a benchmark problem. In [7], a neural network assisted model reference control for this model is studied. Feldkamp et al [8] apply dynamic gradient methods using neural networks for control purpose, and Gorinevsky [9] studies the same benchmark problem using affine radial basis function network architecture.

The process is a tank containing water, nutrients, and biological cells ( $x_1$ ) where the cells mix with nutrients ( $x_2$ ). The volume in the tank is maintained at a constant level by removing tank contents at a rate equal to the incoming rate which is denoted by  $u$ . This rate is called the flow rate and is the variable by which the bioreactor is controlled. The bioreactor control problem is to maintain the cell concentration ( $x_1$ ) at a desired level by appropriately altering the inflow rate ( $u$ ). The plant is kept in an ordinary feedback loop as illustrated in Fig. 1, and the method discussed is applied as the control scheme. The observed state variable is corrupted by a Gaussian distributed random noise having zero mean and variance given in Table 1 for each application. Furthermore, we set  $\delta = 0.25$  for the sign function smoothing parameter. The continuous-time equations of the plant dynamics are given by (18) and (19).

$$\dot{x}_1 = -x_1 u + x_1(1-x_2)e^{x_2/\kappa_1} \tag{18}$$

$$\dot{x}_2 = -x_2 u + x_1(1-x_2)e^{x_2/\kappa_1} \frac{1+\kappa_2}{1+\kappa_2-x_2} \tag{19}$$

where  $0 < x_1, x_2 < 1$ ,  $0 < u < 2$ ,  $\kappa_1 = 0.48$  (nutrient inhibition parameter) and  $\kappa_2 = 0.02$  (growth rate parameter). Furthermore, the initial values are random variables uniformly distributed over the intervals  $0.1086 < x_1(0) < 0.1328$  and  $0.7921 < x_2(0) < 0.9681$ . The control system has been simulated with a step size of 0.01 sec, and the control period ( $T_s$ ) has been set to 0.5 sec [6]. The bioreactor is a challenging control problem for several reasons. Although the task involves few variables and is easily simulated, its nonlinearity makes it difficult to control. For example, small changes in the values of the parameters can cause the bioreactor to become unstable. The issues of delay, nonlinearity, instability and limit cycles can be studied with the bioreactor control problem. Additionally, significant delays exist between changes in flow rate and the response in cell concentration [6]. Based on the parameter settings given in the second column of Table 1, the results obtained are given in Fig. 4.

In Fig. 4(a), the reference signal ( $r$ , dashed line) and the cell concentration ( $x_1$ , solid line) are illustrated. Especially after the sharp transitions in the reference signal, a fast transient is observed in the cell concentration, and the steady state is reached quickly. The behavior of control input and the nutrients are illustrated Fig. 4(b) and (c) respectively. It is apparent that the constraints of the control problem are met. Fig. 4(d) shows the time evolution of the quantity  $(\underline{\Omega}_k^T \underline{\Omega}_{k+1}) / (\underline{\Omega}_k^T \underline{\Omega}_k)$ . During the simulations, its minimal value is observed as 0.2464, which confirms the analysis presented in the previous section.

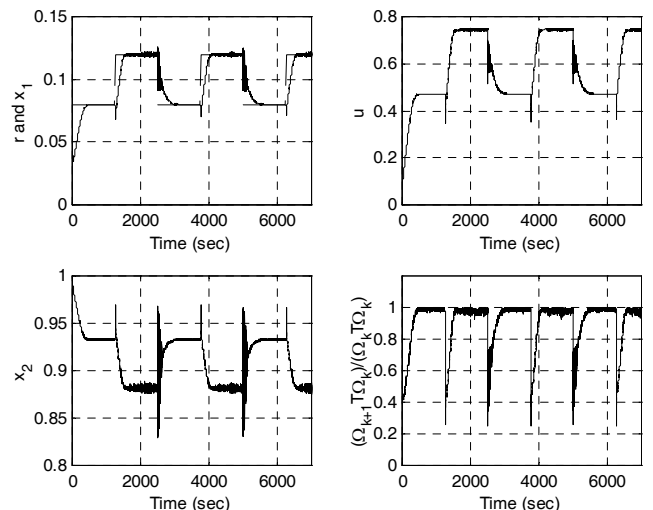


Fig. 4. Simulation Results

- The method discussed does not require the detailed mathematical model of the plant under control, while most control schemes do.

- The observed response quickly converges to the desired levels.
- The duration of the considered sampling periods in the studied example is feasible enough for data acquisition and control evaluation.
- The algorithm is observed to be robust against disturbances, insensitive to parameter variations and is able to compensate large initial errors.
- The computational burden of the algorithm is highly affordable. For the two input single output fuzzy controller having nine rules ( $R=9$ ) in the rule base, a forward pass through the controller requires 57 floating point operations per control period (flops/ $T_s$ ) and 12 comparisons. The tuning of the parameter vector with (12) requires 56 flops/ $T_s$ , which strongly emphasizes the viability of the algorithm. A further attractiveness of the approach is that as the number of rules increases, the computational burden due to the tuning law of (12) increases linearly, and its exact contribution is equal to  $6R+2$  flops/ $T_s$ . Therefore the complexity due to the tuning mechanism is  $O(R)$  and the scheme can be implemented even with average speed microcontrollers.

Table 1. Simulation Data

Controller input vector	$e_{1k}=x_1(k)-r(k)$ $e_{2k}=e_{1k}-e_{1k-1}$
Control period	$T_s=0.5$ sec
Simulation step size	0.01 sec
Uncertainty bound	$\gamma=0.001$
Initial conditions	$e_{1k}(k=0)=0.0276$ $e_{2k}(k=0)=-0.0022$ $x_{1k}(k=0)=0.109837$ $x_{2k}(k=0)=0.800891$
Noise variance	$6.4923e-008$
Noise peak value with probability $\approx 1$	$1e-3$
sgn(.) function Smoothing Parameter	$\delta=0.05$
Switching function	$s_k=e_{1k}$
Membership function parameters	$L_1=0.2$ $L_2=0.2$
Reaching law parameters	$\lambda_1=1.98$ $\lambda_2=10$
Initial controller parameters	$\beta_k(k=0)=0.1 \mathbf{1}_{R \times 1}$

the extraction of the equivalent control error, and the utilization of it in a new parameter tuning scheme. The controller is a standard fuzzy system, which has two inputs and single output. The adjustable parameters are those effective in the defuzzification stage. The proposed technique has been tested on the dynamic model of a biochemical process, the governing equations of which are assumed to be unknown but belong to a particular class. The results obtained through the simulations have shown that a good tracking performance can be achieved in under the presence of time-varying parameters, disturbances and large initial errors. The method has been shown to be computationally efficient for real time control applications.

The future research on this topic aims both to prove that an upper bound for the Euclidean norm of the adjustable parameter vector exists and to demonstrate that a hitting is guaranteed to occur in finite time.

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### 4 Conclusions

A novel method for discrete time fuzzy sliding mode control is studied in this paper. The method is based on