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REMOVING THE DELAYED INTERFERENCE OF CONTROL INPUT IN ROBUST SETPOINT TRACKING

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ABSTRACT

This paper presents a method to design feedback controllers where the plant under control is linear and uncertain and its nominal representation is available. The controller handling the effects of the uncertainties has two zeros, two poles and a gain. The frequency domain behavior of the necessary controller is obtained and the data is approximated by the chosen controller structure. The results are justified through some simulations. The relevance of the current paper to the aerospace engineering is the necessity to robust control schemes for every air or space vehicle whose motion is subject to uncertain dynamics and disturbances. The contribution of the current paper is to develop an effective method to obtain a linear controller to alleviate the difficulties caused by uncertainty terms.

INTRODUCTION

In aerospace systems, particularly for telerobotics applications, the command signal sent from the ground control station arrives at the destination after a significant time delay, which depends on the physical distance between the source and the target, and the hardware used [Safaric et al, 1999]. The delay in the control signal is a significant drawback that entails a careful study to explain the stability and performance issues. This paper considers a unit mass and single dimension with single actuator. The resulting model is a double integrator with delayed input. Two examples are studied, namely, a delayed control input with 1 second of time delay in the first case. The second case considers a time delay of 0.1 seconds with a reflection term bringing a delay of 1 second in the control input line. The present study considers a nominal control loop and a nominal controller augmented with a secondary loop to handle the uncertainties introduced by the delay terms. The importance of the current work is to understand the two loop structure of the control system and necessary steps to meet the design specifications.

The technical motivation of this study is to develop a scheme to handle uncertainties. In the literature, numerous techniques with different sets of performance specifications and structural properties have been studied. The goal in each has been to observe a desired behavior in some optimal sense. In each case, a model of the plant is either derived or available. This model is called the nominal model and the design is primarily based on the information contained in the nominal model. The field of robust control offers solutions to cases where there are deviations from the nominal model and the obtained controllers are applicable to plants that belong to particular classes, [Doyle et al, 1990; Foias et al, 1996]. What motivates us here is to observe robustness in the presence of uncertainties yet the controller based on the nominal system model is responsible for the nominal response and the deviations caused by the uncertainties are handled by a secondary controller. The latter admits the difference in between the nominal model response and the uncertain plant response and output is subtracted from the nominal control signal. The system structure is shown in Figure 1. The shown connectivity addresses the aforementioned design philosophy and the question is how to obtain the secondary controller. We study time delays as they have significant consequences in stability and performance.

Time delay systems have been studied by many researchers in the past and there is a growing interest to the topic as the application areas cover networked control to remotely controlled spacecraft systems, [Zhong, 2006], where the essential issues in the design phase are discussed in detail.

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Among those, most critical one is the stability, which can make the closed loop system unstable [Gu et al, 2003]. The literature on time delay systems and control has a certain degree of maturity and the interested reader is referred to the references in the cited works. The focus of the current work is to describe a system structure to handle uncertainties appropriately. The structure needs a nominal model and this aspect makes it similar to internal model control (IMC) [Jin et al, 2014] and model predictive control (MPC) [Zhang et al, 2014] approaches. Indeed, neither IMC nor MPC has a second control loop to improve the behavior. The proposed structure is therefore an alternative approach to the available techniques. Especially the need of optimization at every control period makes MPC disadvantageous compared to the current approach.

This paper is organized as follows: The second section describes the feedback loop, the assumptions and the main contribution of the paper. In the third section, we consider the first numerical example, which is a double integrator with a single term multiplicative uncertainty. In the fourth section, an intermediate problem, i.e. approximating the desired controller is described. The fifth section presents the details of the simulations with the second example, which contains two different delay terms. The concluding remarks are given at the end of the paper.

STRUCTURE OF THE FEEDBACK LOOP, THE ASSUMPTIONS AND THE CONTRIBUTION

The structure of the control system is shown in Figure 1, where the nominal plant model is denoted by P_n and C_1 is the controller designed for the control of the nominal system. For the loop formed by nominal plant and the nominal controller, we have the following relation.

$$Y_n(s) = \frac{P_n(s)C_1(s)}{1+P_n(s)C_1(s)} R_1(s) := T_1(s)R_1(s) \tag{1}$$

Assumption 1. The nominal representation of the plant is available and the nominal process controller denoted by $C_1(s)$ is designed in such a way that the closed loop transfer function $T_1(s)$ is stable and it meets the desired performance specifications if there are no uncertainties.

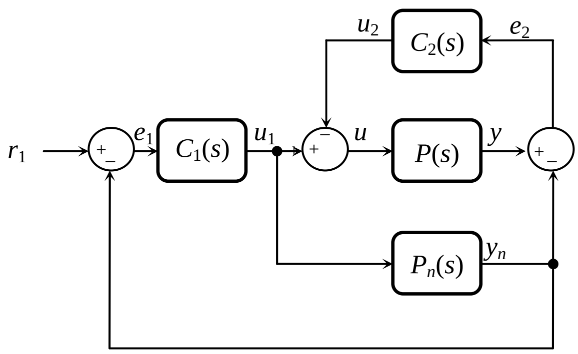

Figure 1: *Block diagram of the control system*

Using the rules of block diagram simplification, it is straightforward to show that the transfer function from $Y(s)$ to $R_1(s)$ is given as below.

$$\frac{Y}{R_1} = \frac{\left(\dfrac{PC_2}{1+PC_2}\right)}{\left(\dfrac{P_nC_2}{1+P_nC_2}\right)} \left(\frac{P_nC_1}{1+P_nC_1}\right) \tag{2}$$

Consider the uncertain plant in the following form, which is a multiplicatively perturbed extension of the nominal plant dynamics.

$$P(s) = \left(1 + \Delta(s)\right) P_n(s) \tag{3}$$

If (3) is inserted into (2), rearranging the terms according to the uncertainty terms simplify and the transfer function from Y to Y_n can be given as in (4).

$$\frac{Y}{Y_n} = \frac{1+\Delta}{1+\frac{P_n C_2}{1+P_n C_2}\Delta} = \frac{1+\Delta(s)}{1+Q(s)\Delta(s)} \tag{4}$$

The expression above can be interpreted as follows. The desired value of $Q(s)$ is unity for all frequencies yet this corresponds to $P=P_n$. The uncertainties affecting the low frequency behavior result in visible deterioration in the closed loop performance and the desired choice for $Q(s)$ is to make it a low pass transfer function. However, looking at the connectivity in Figure 1, the compensator $C_2(s)$ addresses the deviations caused by the uncertainties, it needs to have a high pass behavior to address the undesired changes.

FIRST NUMERICAL EXAMPLE

As the first numerical example, we consider

$$P_n(s) = \frac{1}{s^2} \tag{5}$$

$$P(s) = \frac{1}{s^2}\left(1 + e^{-\tau s}\right), \quad \tau = 1 \tag{6}$$

The nominal plant model is a double integrator. The differential equation in time domain is $\ddot{y}_n(t) = u(t)$. However, the uncertain system modifies this dynamics as $\ddot{y}(t) = u(t) + u(t-\tau)$, where the delayed value of the input is added to itself. This choice is deliberate as the aerospace systems are frequently subject to such delays caused by interference or channel impurities. With this plant, choose the desired low pass transfer function as

$$H_{LP}(s) := \frac{a^m}{(s+a)^m}, \; a > 0, \; m \in \Re^+ \tag{7}$$

and write the transfer function Q by substituting the known variables and equate this to H_{LP} as

$$Q = \frac{P_n C_2}{1+P_n C_2} = \frac{C_2/s^2}{1+C_2/s^2} = \frac{C_2}{s^2 + C_2} := H_{LP}(s) = \frac{a^m}{(s+a)^m} \tag{8}$$

This yields

$$C_2(s) = \frac{H_{LP}(s)}{1-H_{LP}(s)}\frac{1}{P_n(s)} = \frac{s^2 a^m}{(s+a)^m - a^m} \tag{9}$$

With $m = 2$, $C_2(s)$ displays a high pass behavior and $Q(s)$ becomes a low pass transfer function. The corresponding $C_2(s)$ is given in (10).

$$C_2(s) = \frac{as}{s+2a} \tag{10}$$

With $m = 1$, $C_2(s)$ again displays a high pass behavior and $Q(s)$ becomes a low pass transfer function and $C_2(s)$ becomes a pure differentiator with gain a, i.e. $C_2(s) = as$.

To study the stability of the closed loop system, write (4) as follows. The open loop transfer function is

$$\frac{Y}{Y_n} = \frac{1+\Delta W_2}{1+\frac{P_n C_2}{1+P_n C_2}\Delta W_2} = \frac{1+e^{-\tau s}}{1+H_{LP}e^{-\tau s}} = \frac{1}{1+H_{LP}e^{-\tau s}} + \frac{e^{-\tau s}}{1+H_{LP}e^{-\tau s}} \tag{11}$$

The transfer function in (11) is always stable with the choice in (7). Regarding the performance, non integer values of m could yield better performance compared to integer m values. In order to synthesize the controller $C_2(s)$, we utilize numerical approximation as explained in the next section.

APPROXIMATING THE DESIRED CONTROLLER

In this paper, we consider the following controller functioning in the place of $C_2(s)$.

$$C_2(s) = K\frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \tag{12}$$

We denote the desired $C_2(s)$ by $C_{2d}(s)$ and define the magnitude and the phase of the desired $C_{2d}(s)$ as follows.

$$C_{2d}(j\omega) = M(\omega)\exp\left(jA(\omega)\right) \tag{13}$$

The numerical values of $M(\omega)$ and $A(\omega)$ will be obtained from the design approach presented in the previous section (See (9)). Denote the frequency by ω and define the following cost function:

$$J = \frac{1}{2}\sum_{\omega\in[\omega_L,\omega_U]}\left(e_M(\omega)^2 + e_A(\omega)^2\right) \tag{14}$$

where ω_L and ω_U are the lower and upper bounds of the considered frequency spectrum, respectively, and we have the following error definitions to compute the cost in (14).

$$e_M(\omega) := M(\omega) - K\frac{\sqrt{\omega^2+z_1^2}\sqrt{\omega^2+z_2^2}}{\sqrt{\omega^2+p_1^2}\sqrt{\omega^2+p_2^2}} \tag{15}$$

$$e_A(\omega) := A(\omega) - \left\{\arctan\left(\frac{\omega}{z_1}\right) + \arctan\left(\frac{\omega}{z_2}\right) - \arctan\left(\frac{\omega}{p_1}\right) - \arctan\left(\frac{\omega}{p_2}\right)\right\} \tag{16}$$

The optimization problem is to find the best z_1, z_2, p_1, p_2 and K parameters that fit the desired magnitude and phase responses. In order to perform this optimization, we chose the Levenberg-Marquardt technique and formulate the following partial derivatives.

$$\frac{\partial e_M(\omega)}{\partial z_1} = -K\frac{\sqrt{\omega^2+z_2^2}}{\sqrt{\omega^2+p_1^2}\sqrt{\omega^2+p_2^2}}\frac{z_1}{\sqrt{\omega^2+z_1^2}} \tag{17}$$

$$\frac{\partial e_M(\omega)}{\partial z_2} = -K\frac{\sqrt{\omega^2+z_1^2}}{\sqrt{\omega^2+p_1^2}\sqrt{\omega^2+p_2^2}}\frac{z_2}{\sqrt{\omega^2+z_2^2}} \tag{18}$$

$$\frac{\partial e_M(\omega)}{\partial p_1} = \frac{Kp_1\sqrt{\omega^2+z_1^2}\sqrt{\omega^2+z_2^2}\sqrt{\omega^2+p_2^2}}{(\omega^2+p_1^2)(\omega^2+p_2^2)\sqrt{\omega^2+p_1^2}} \tag{19}$$

$$\frac{\partial e_M(\omega)}{\partial p_2} = \frac{Kp_2\sqrt{\omega^2+z_1^2}\sqrt{\omega^2+z_2^2}\sqrt{\omega^2+p_1^2}}{(\omega^2+p_1^2)(\omega^2+p_2^2)\sqrt{\omega^2+p_2^2}} \tag{20}$$

$$\frac{\partial e_M(\omega)}{\partial K} = -\frac{\sqrt{\omega^2+z_1^2}\sqrt{\omega^2+z_2^2}}{\sqrt{\omega^2+p_1^2}\sqrt{\omega^2+p_2^2}} \tag{21}$$

$$\frac{\partial e_A(\omega)}{\partial z_i} = \left(\frac{1}{\omega}\right)\frac{\left(\frac{\omega}{z_i}\right)^2}{1+\left(\frac{\omega}{z_i}\right)^2}, \quad i=1,2 \tag{22}$$

$$\frac{\partial e_A(\omega)}{\partial p_i} = -\left(\frac{1}{\omega}\right)\frac{\left(\frac{\omega}{p_i}\right)^2}{1+\left(\frac{\omega}{p_i}\right)^2}, \quad i=1,2 \tag{23}$$

$$\frac{\partial e_A(\omega)}{\partial K} = 0 \tag{24}$$

Considering the finite set of frequencies, i.e. $\omega := \omega_1, \omega_2, \ldots \omega_N$, the partial derivatives above yield vectors of dimensions $N\times1$ and the Jacobian matrix of Levenberg-Marquardt algorithm is constructed as given below.

$$\Omega(t) := \begin{bmatrix} \frac{\partial e_M}{\partial K} & \frac{\partial e_M}{\partial z_1} & \frac{\partial e_M}{\partial z_2} & \frac{\partial e_M}{\partial p_1} & \frac{\partial e_M}{\partial p_2} \\ \frac{\partial e_A}{\partial K} & \frac{\partial e_A}{\partial z_1} & \frac{\partial e_A}{\partial z_2} & \frac{\partial e_A}{\partial p_1} & \frac{\partial e_A}{\partial p_2} \end{bmatrix}_{2N\times5} \tag{25}$$

$$\begin{bmatrix} K(t+1) \\ z_1(t+1) \\ z_2(t+1) \\ p_1(t+1) \\ p_2(t+1) \end{bmatrix} = \begin{bmatrix} K(t) \\ z_1(t) \\ z_2(t) \\ p_1(t) \\ p_2(t) \end{bmatrix} - \left(\mu I_{5\times5} + \Omega(t)^T\Omega(t)\right)^{-1}\Omega(t)^T E(t), \quad \mu>0 \tag{26}$$

where

$$E(t) := \begin{bmatrix} e_M(\omega) \\ e_A(\omega) \end{bmatrix} \tag{26}$$

The rule in the above behaves like the Gauss-Newton algorithm for small μ, whereas it is like gradient descent for large μ. The algorithm is started with small random controller gains and it finds the best matching parameters quickly. In this paper $\mu = 10$ is selected in all simulations.

Since the optimization process cannot guarantee $z_i > 0$ or $p_i > 0$, during the iterations, a stability check condition is inserted. If any one of the zeros or poles of $C_2(s)$ tend to be unstable, that variable is reinitialized randomly to a very small positive value and the optimization is continued with the new value of the parameter of interest

SIMULATION STUDIES

First Example: Double Integrator

A number of cases have been studied for the nominal plant in (5) and the uncertain plant in (6). The controller for the nominal plant has been designed as given in (27), and used in all simulations.

$$C_1(s) = 3 + \frac{1}{s} + \frac{12s}{1 + s/100} = \frac{3s^2 + 1501s + 100}{s(s+100)} \quad (27)$$

The response of the nominal plant model with the above $C_1(s)$ is shown together with the response of the uncertain plant and the nominal controller in Figure 2, where it is seen that the desired response is fairly smooth yet the uncertain plant response exhibits undesired oscillations The goal of adding the loop with $C_2(s)$ is to suppress these oscillations to an admissible extent.

As the a parameter, we considered a={0.5, 1, 2, 4}, and as the m parameter, we considered m={1, 1.1, 1.5, 1.9, 2, 2.1}. The combinations have been studied exhaustively and the results for $a = 0.5$ are given in Figures 3-4, those for $a = 1$ are given in Figures 5-6, for $a = 2$ are given in Figures 7-8, and for $a = 4$ are given in Figures 9-10. Since Q needs to be a low pass type transfer function, according to the presented sets of figures, as a increases, gain margin (GM) decreases and oscillations become visible. For small a, the convergence is considerably quick and the initial overshoot is large. In Figures 3, 5, 7, 9, the response corresponding to $m = 1$ is drawn by a thick curve. For this choice, $C_2(s)$ approximates to the pure differentiator, given as $C_{2d}(s) = as$.

Second Example

In the second example, we consider the same nominal plant model but the uncertain system now contains another term as described below.

$$P(s) = \frac{1}{s^2}\left(1 + e^{-\tau s} + e^{-0.1\tau s}\right), \quad \tau = 1 \quad (28)$$

The nominal controller cannot stabilize the closed loop, the results for a = 0.5 are as seen in Figure 11. When the proposed scheme is used, the response in time domain is as seen in Figure 12, where the system in (28) converges the desired setpoint. The Nyquist plot for this case is shown in Figure 13 and the output of the proposed controller is shown in Figure 14. Bounded nature of the produced control signal is a prominent feature. The time domain results for $a = 1$, corresponding Nyquist plots and the produced control signals are shown, respectively, in Figures 15-17. The results in the same order are shown in Figures 18-20 for $a = 2$. In all cases considered, the output of the proposed controller remained bounded and the system followed the reference signal with zero error.

The interpretation of the transfer function Y/Y_n for a general case is illustrated in Figure 21, where it is seen that the transfer functions in the numerator and the denominator of (2) generate two triangles in the Nyquist plot. The uncertainty is bounded and the feedback connectivity in Figure 1 results in the ratio in (2). Clearly in Figure 21, it is possible to apply PID based design and tuning rules as the frequency domain picture is in compliance with classical approaches.

CONCLUSIONS

This paper presents a method to obtain the desired frequency response of an auxiliary controller to handle the deviations from the nominal model response. The approximation to the desired controller is achieved using Levenberg-Marquardt technique. Both the magnitude and the phase values are forced to approximate the desired frequency response data and the obtained controller is a real rational transfer function with two zeros, two poles and a gain, all of which are adjustable. Once fixed, the controller is installed into the closed loop system and tested. The proposed approach enables the designer to study non-integer values of the alternatives and this is an important design flexibility offered to the designer. However, the controller needs the inverse of the nominal system, which is a constraint forcing to study plants with no right half plane zeros.

Motivation of the study was to stabilize a remote system that is subject to delays in the input channel, like what occurs in telerobotic applications. The nominal plant chosen is a double integrator, which represents a mechanical system dynamics and reference signal is a constant setpoint. The delays in the input channel adversely affect the performance and it is shown by a series of simulations and two different cases that the proposed technique enables the designer to obtain a good alternative to suppress the effects of time delays.

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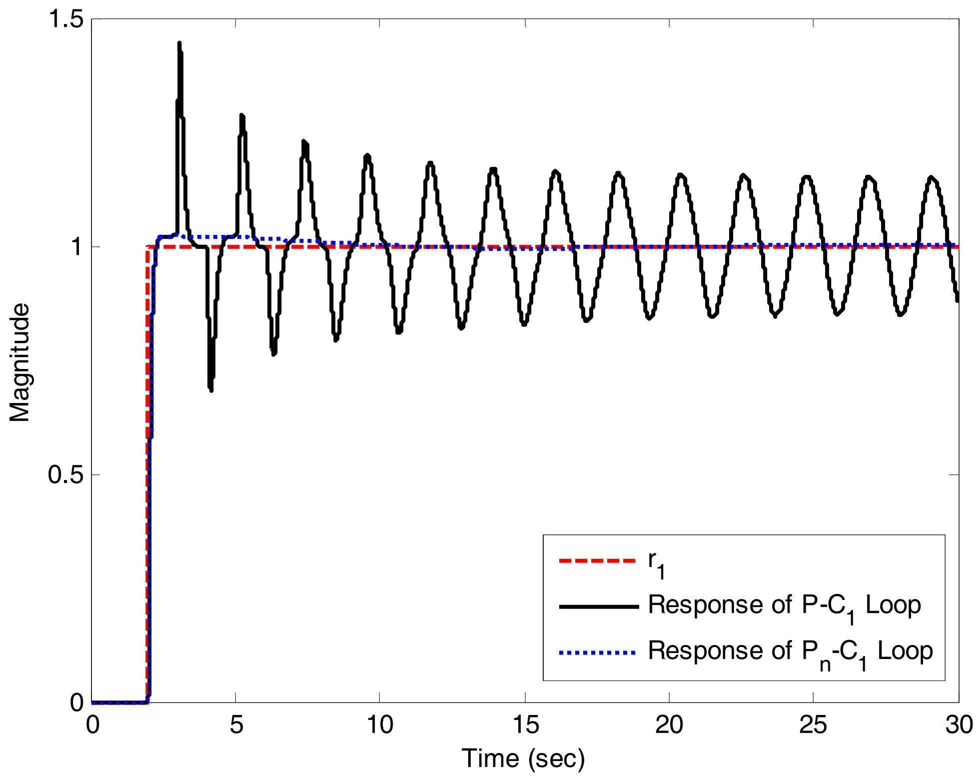

Figure 2. *Response of the nominal plant and nominal controller loop (dotted) and the response of the uncertain plant and nominal controller loop (solid)*

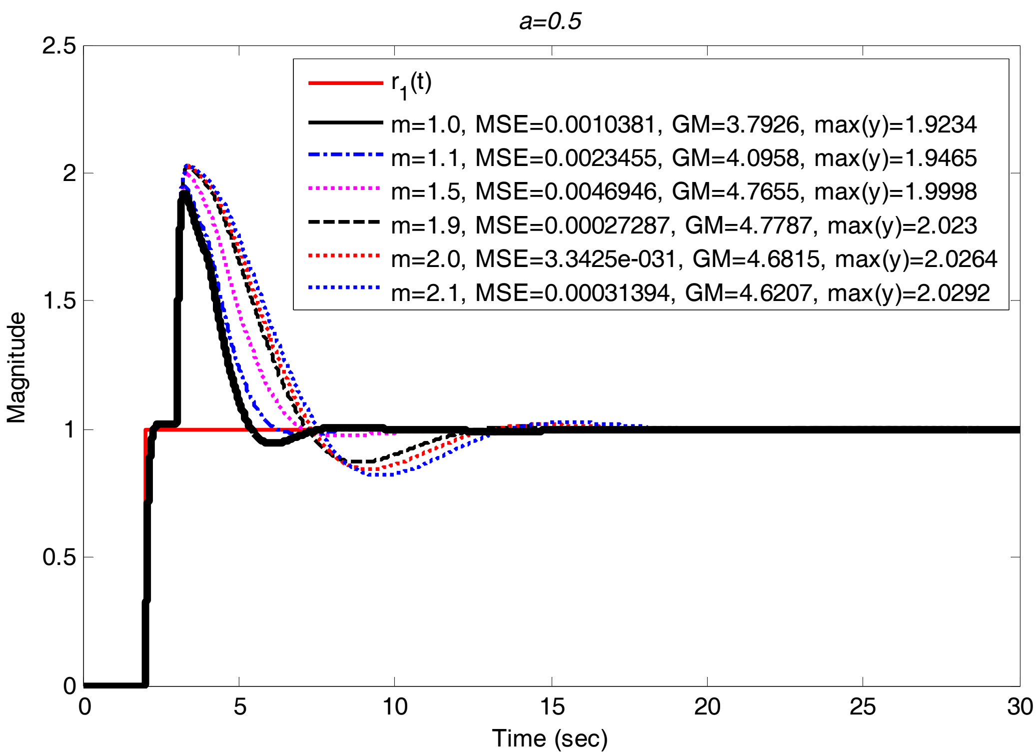

Figure 3. *Time domain results for* a=0.5

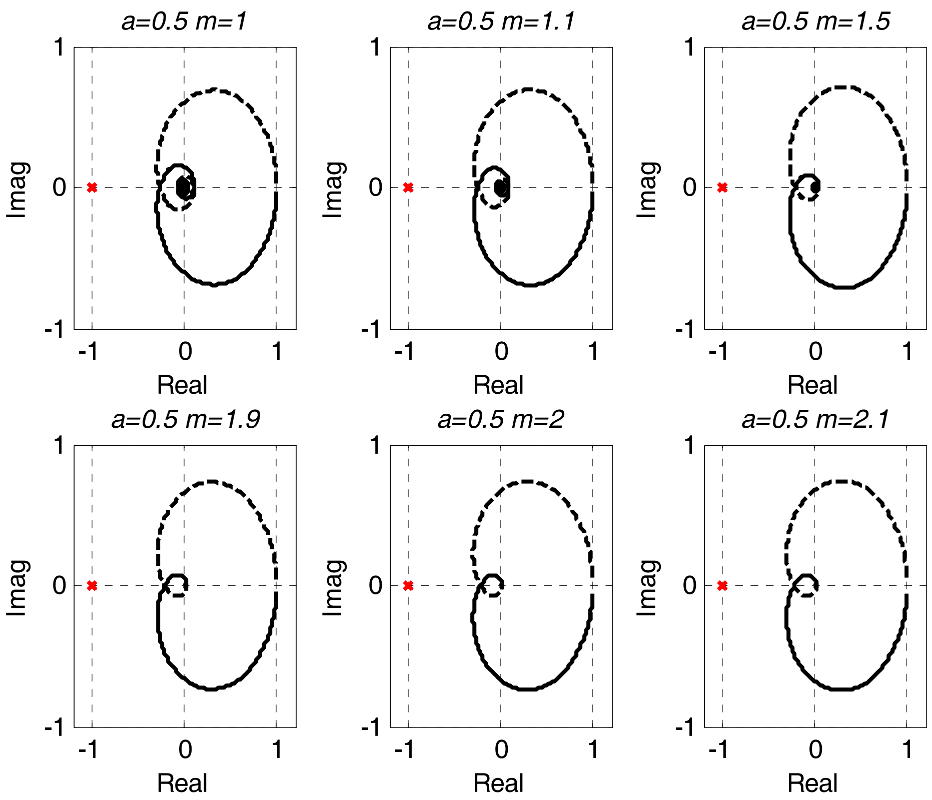

Figure 4. *Nyquist plot of* $Q(j\omega)\exp(-\tau j\omega)$ *for* a=0.5

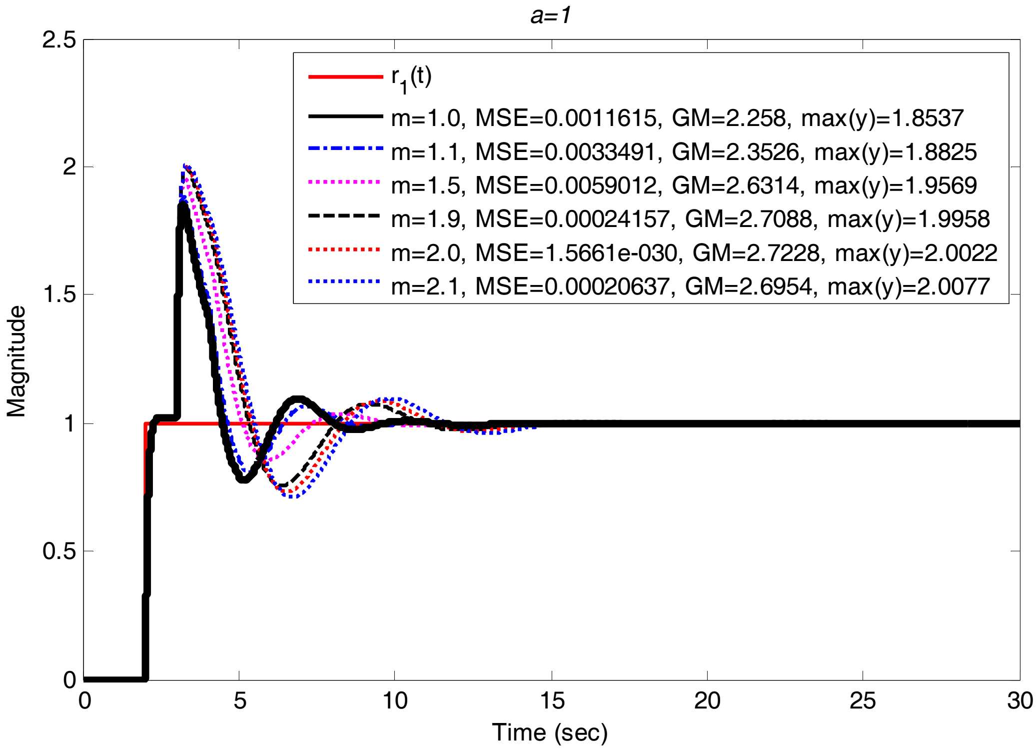

Figure 5. *Time domain results for* a=1

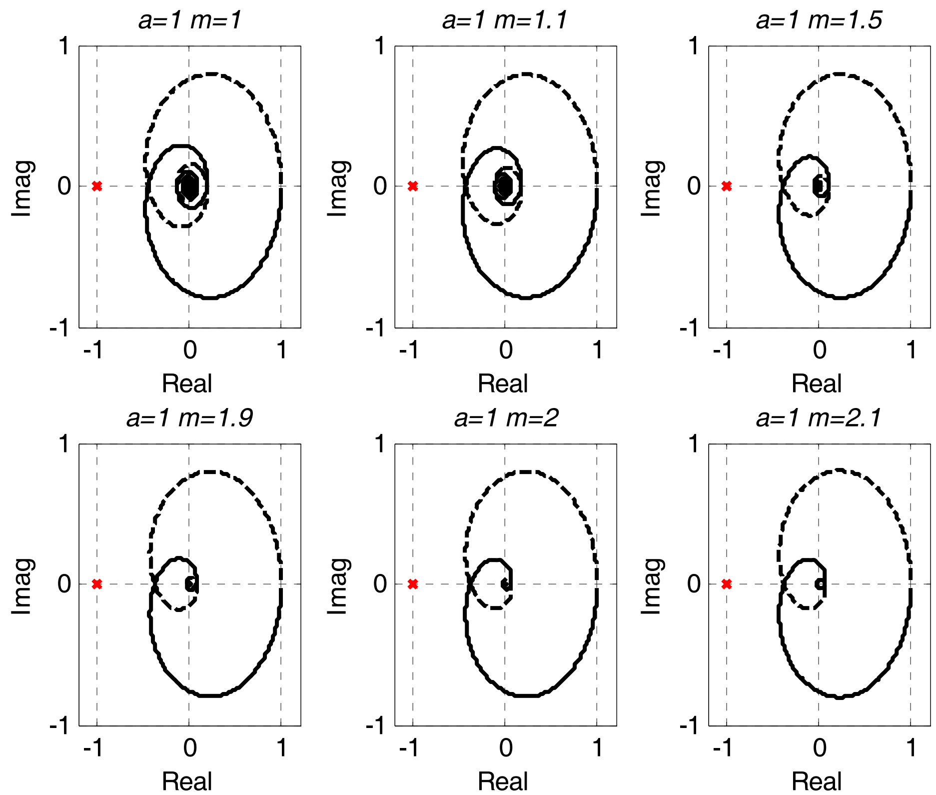

Figure 6. *Nyquist plot of* $Q(j\omega)\exp(-\tau j\omega)$ *for* a=1

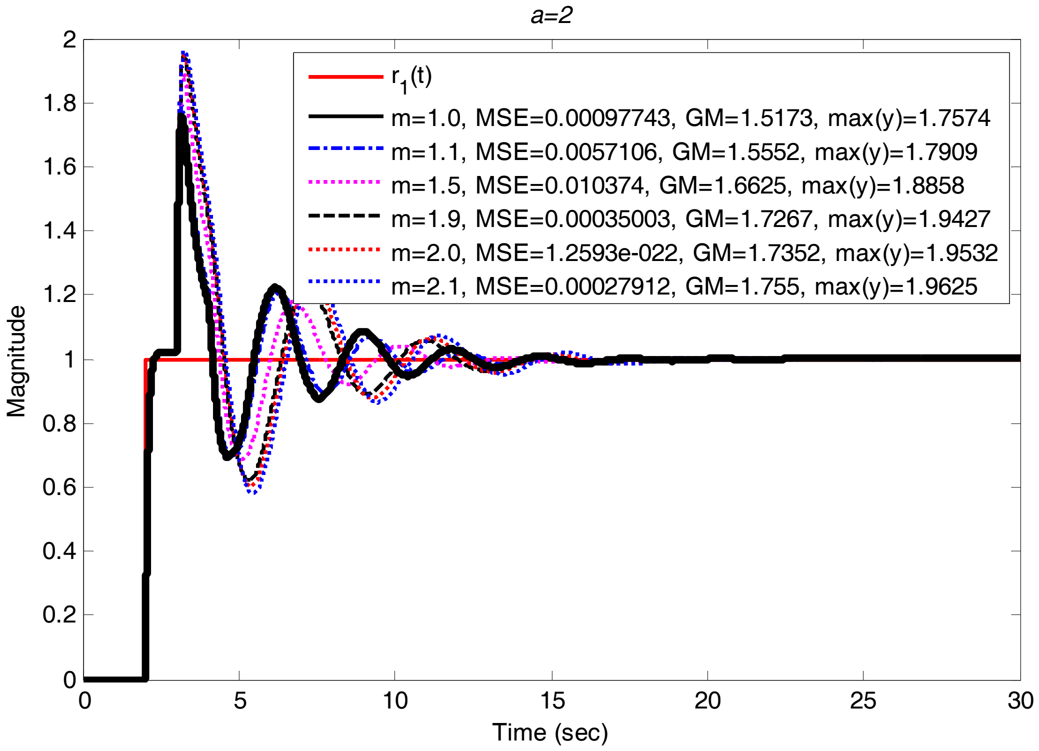

Figure 7. *Time domain results for* a=2

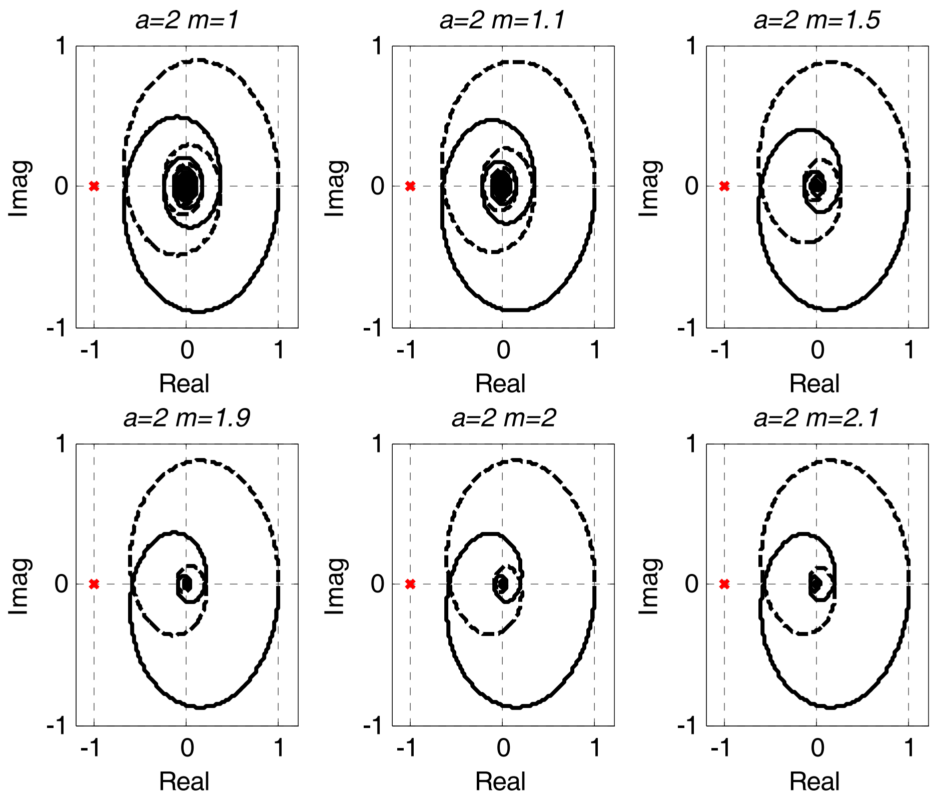

Figure 8. *Nyquist plot of* $Q(j\omega)\exp(-\tau j\omega)$ *for* a=2

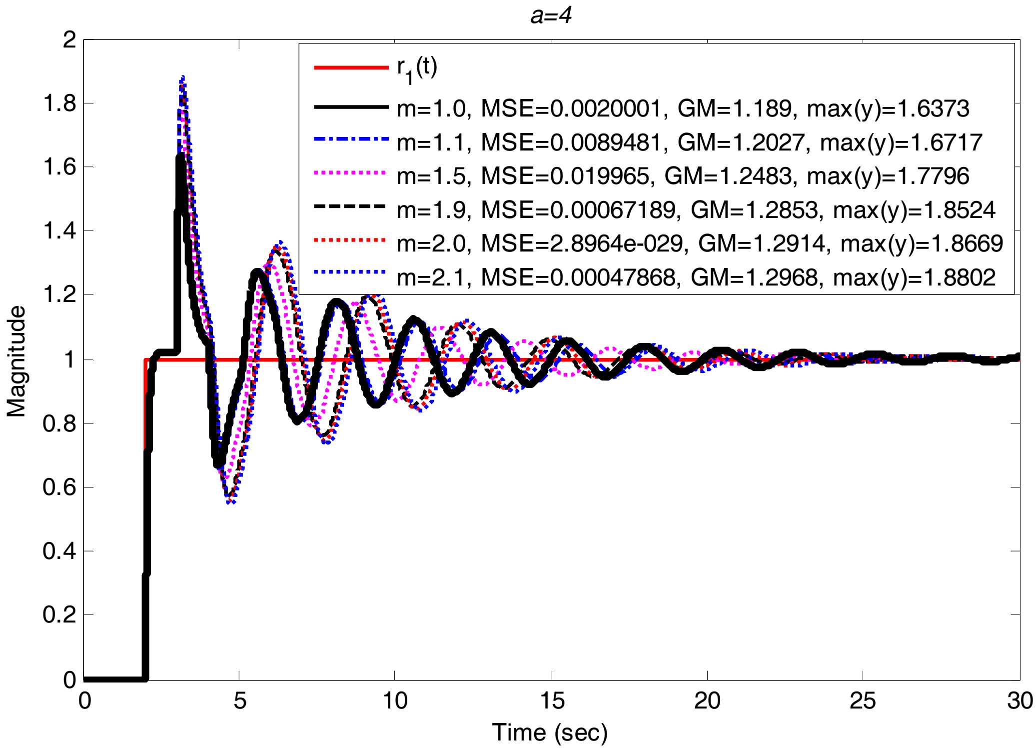

Figure 9. *Time domain results for* a=4

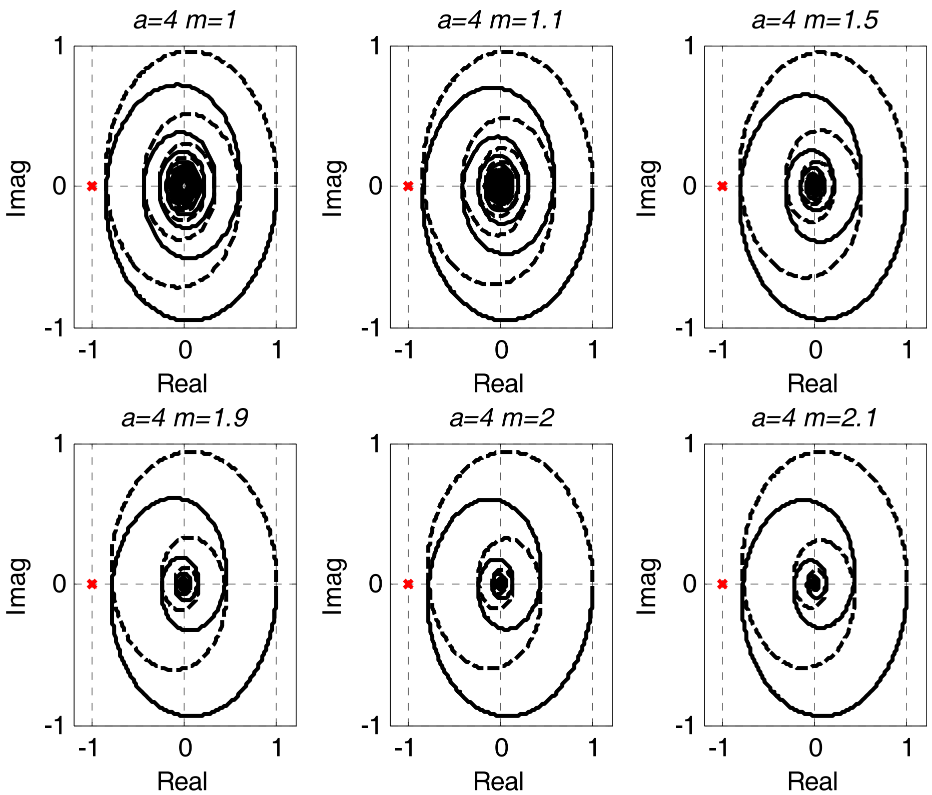

Figure 10. *Nyquist plot of* $Q(j\omega)\exp(-\tau j\omega)$ *for* a=4

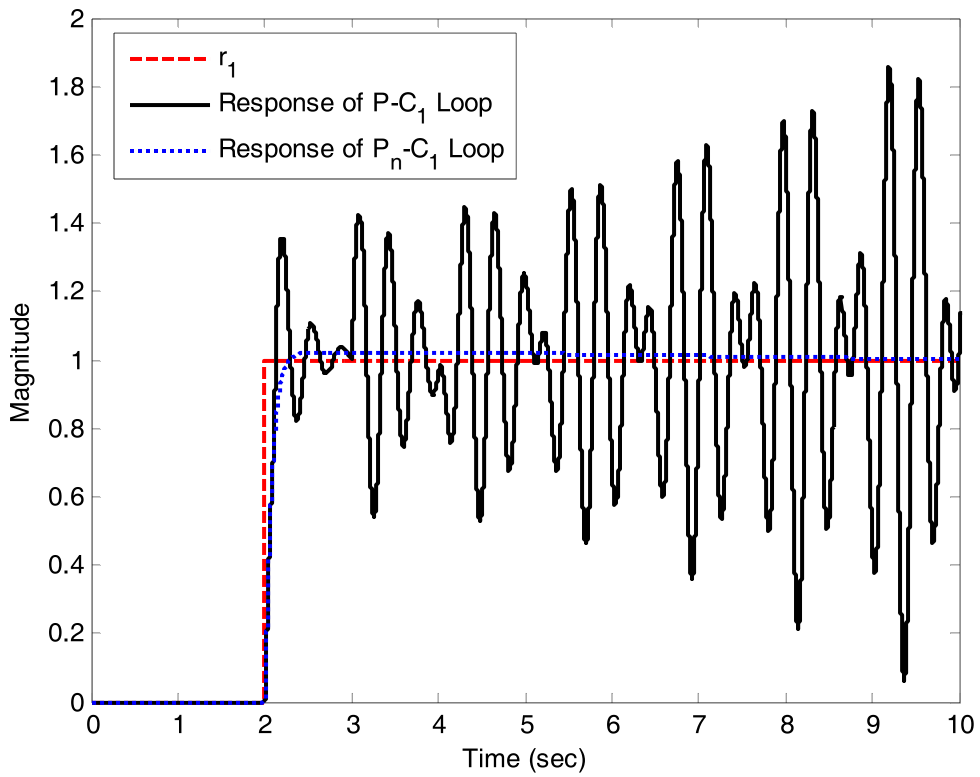

Figure 11. *Response of the nominal system (dotted) and the response of the uncertain plant with nominal controller (solid)*

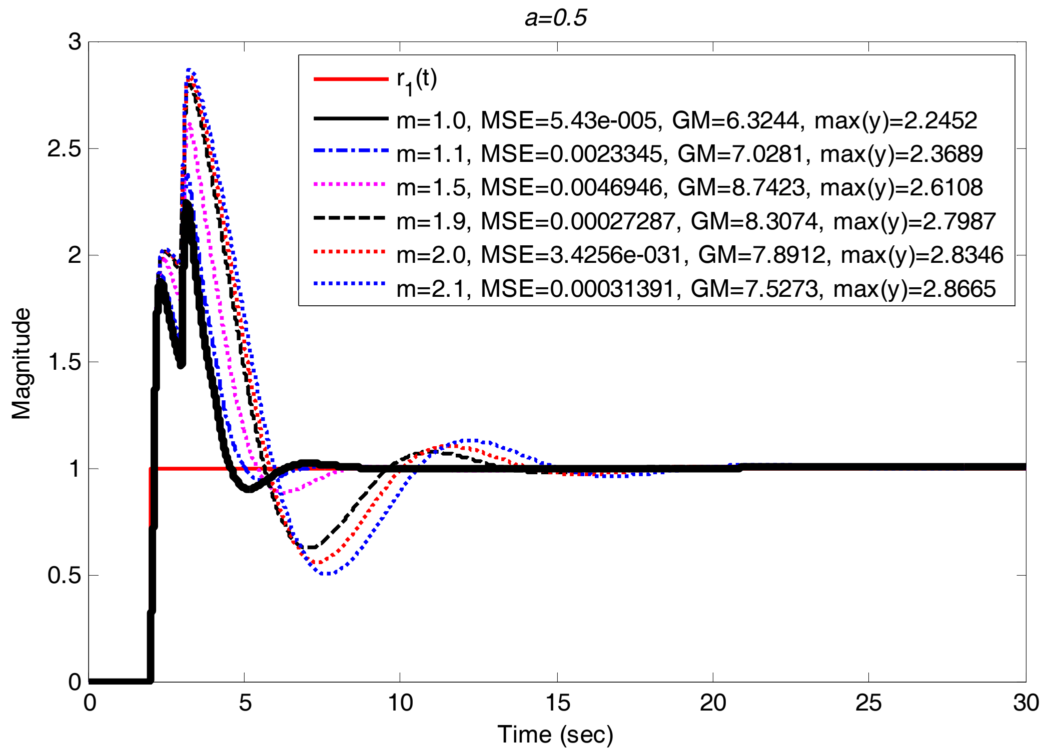

Figure 12. *Time domain results for* a=0.5

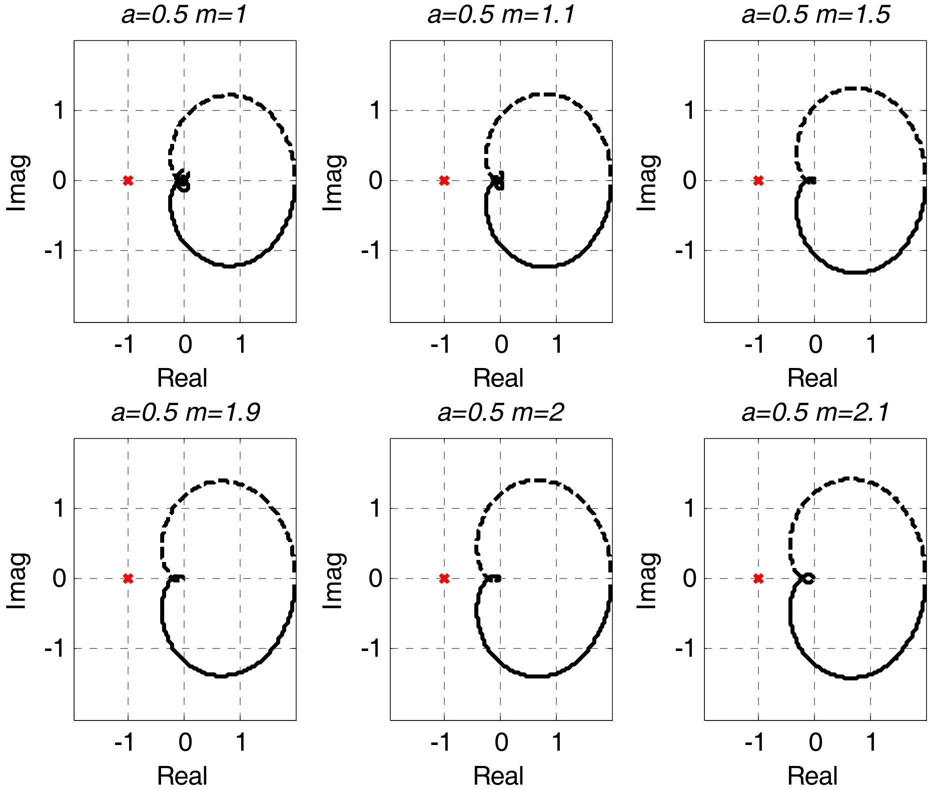

Figure 13. *Nyquist plot of* $Q(j\omega)\left(\exp(-\tau j\omega)+\exp(-0.1\tau j\omega)\right)$ *for* a=0.5

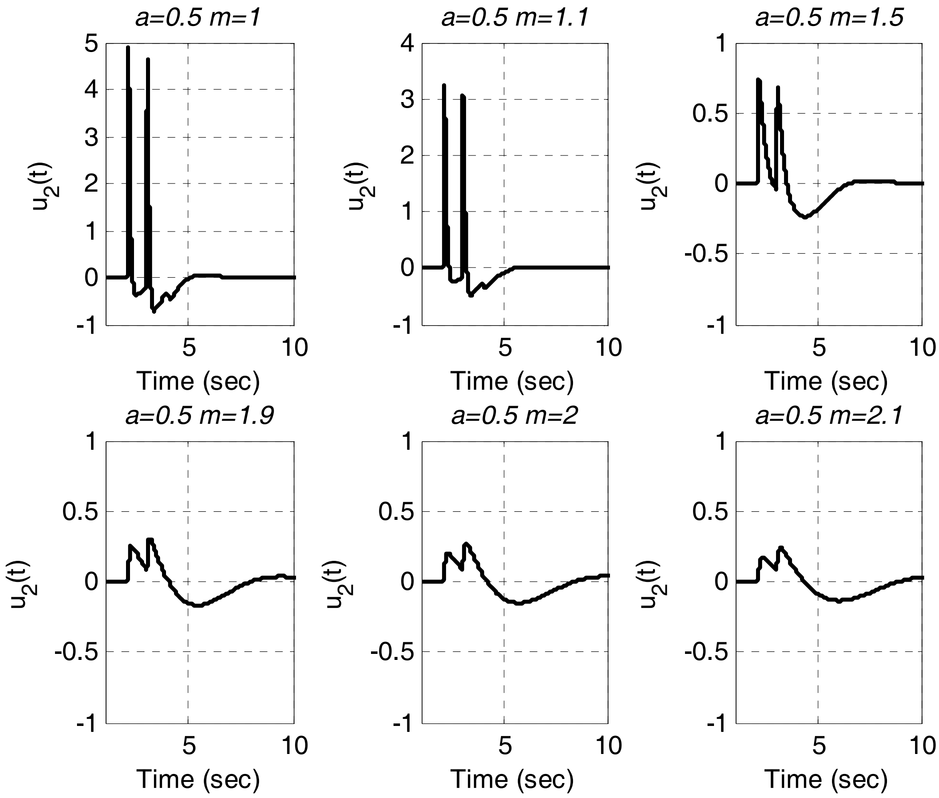

Figure 14. *Contribution of the proposed controller for* a=0.5

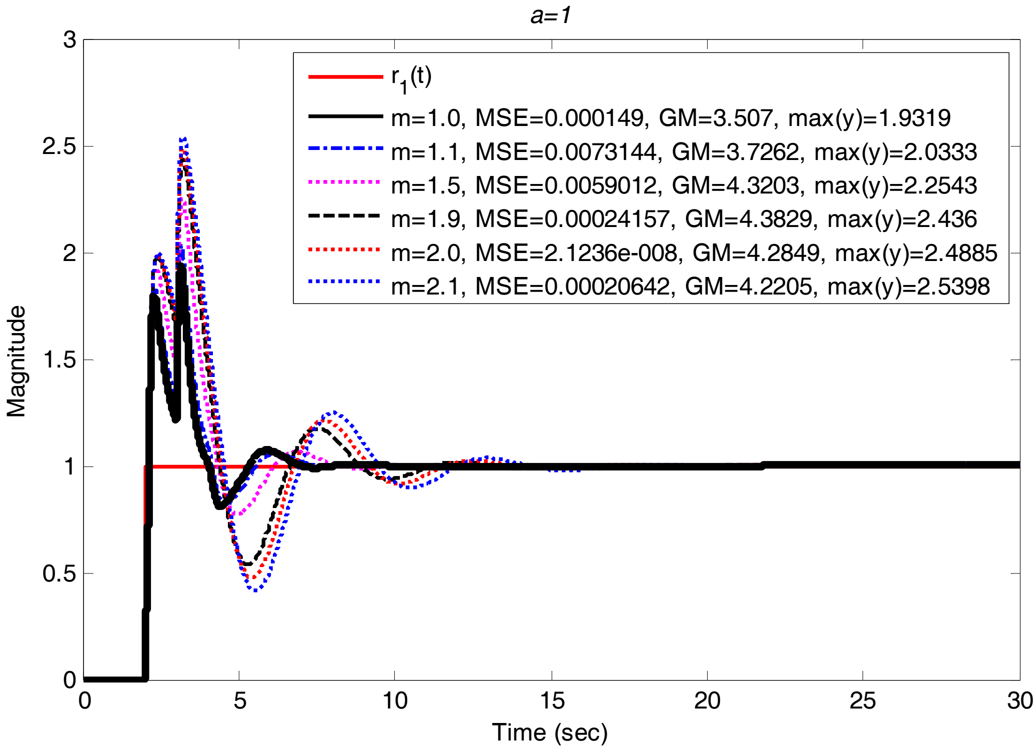

Figure 15. *Time domain results for for* a=1

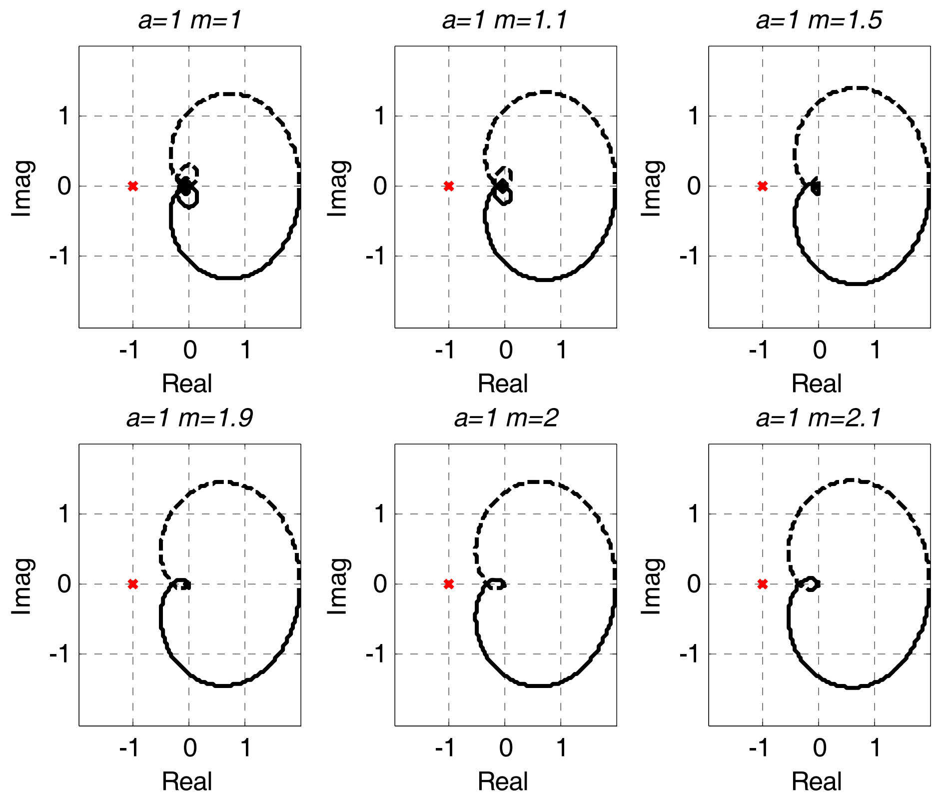

Figure 16. *Nyquist plot of* $Q(j\omega)\left(\exp(-\tau j\omega)+\exp(-0.1\tau j\omega)\right)$ *for* a=1

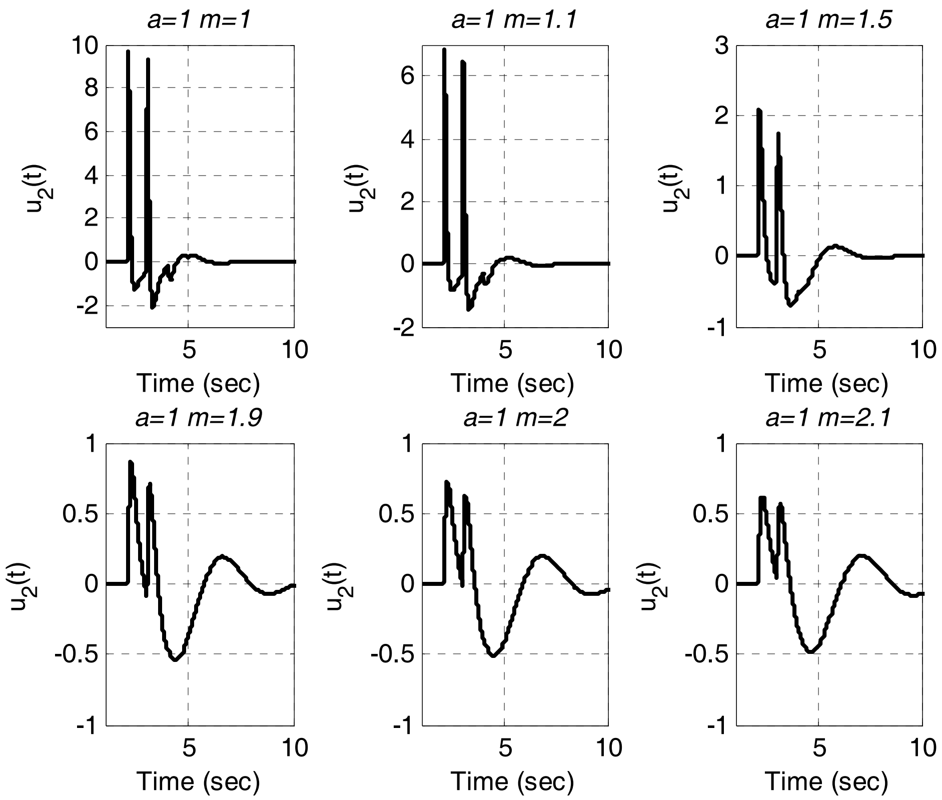

Figure 17. *Contribution of the proposed controller for* a=1

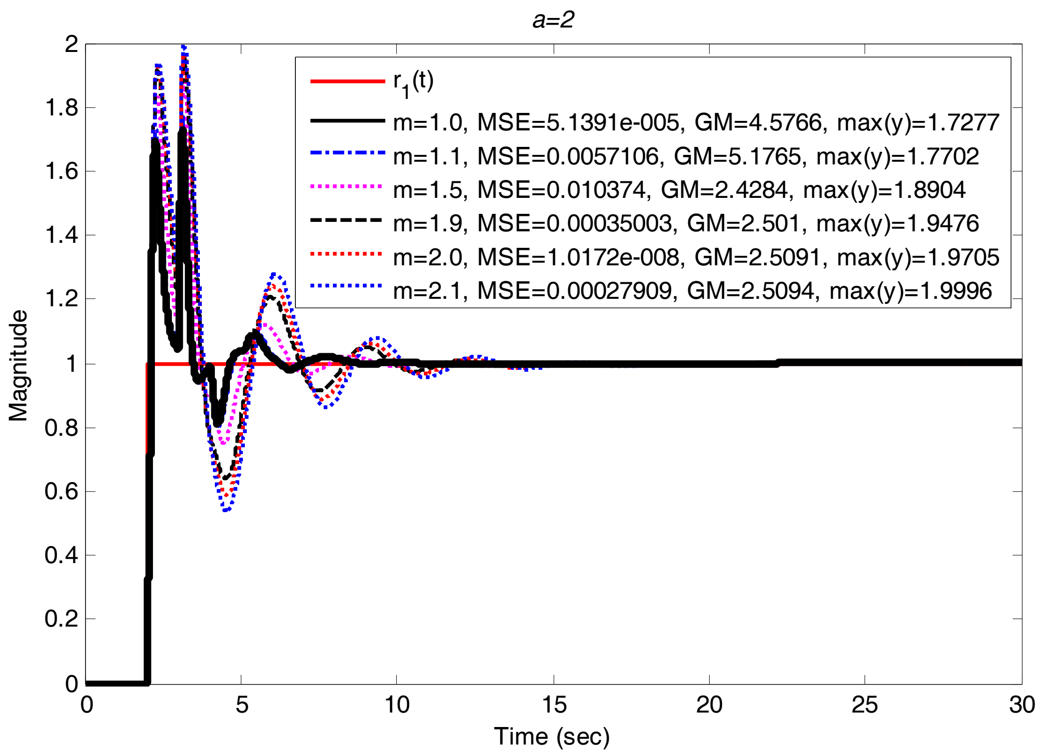

Figure 18. *Time domain results for* a=2

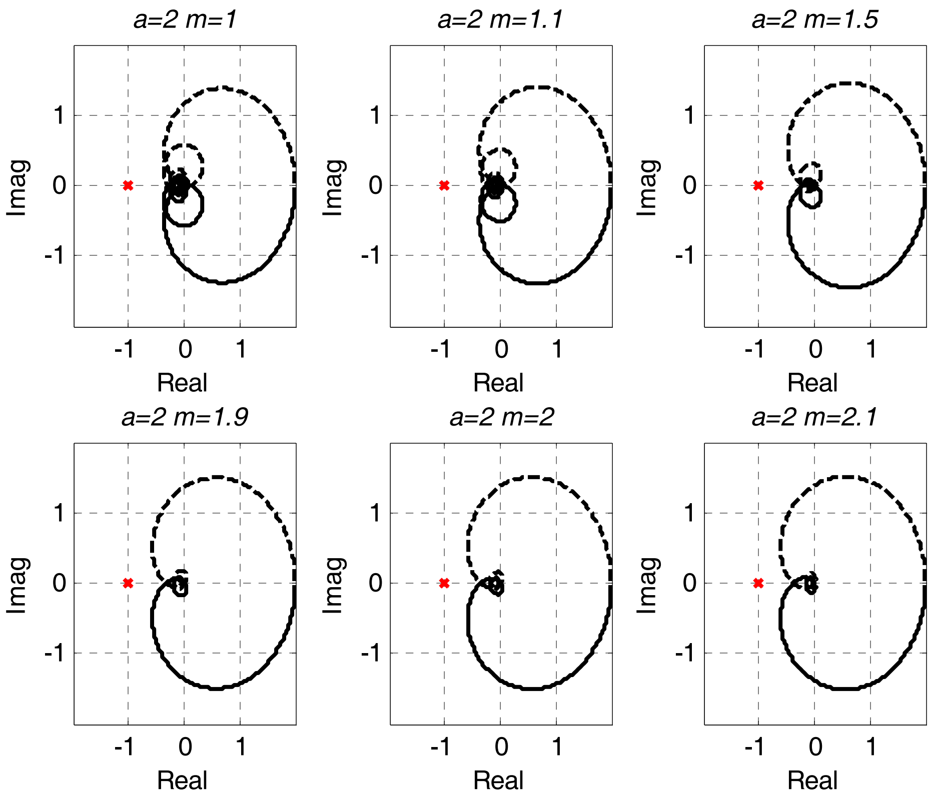

Figure 19. *Nyquist plot of* $Q(j\omega)\left(\exp(-\tau j\omega)+\exp(-0.1\tau j\omega)\right)$ *for* a=2

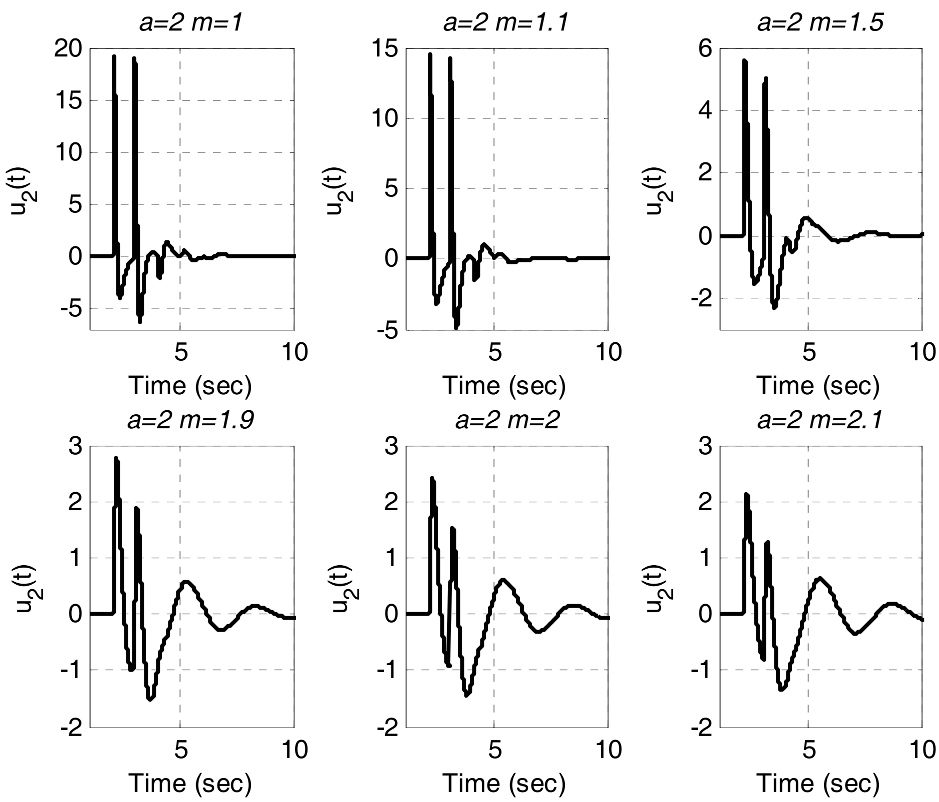

Figure 20. *Contribution of the proposed controller for* a=2

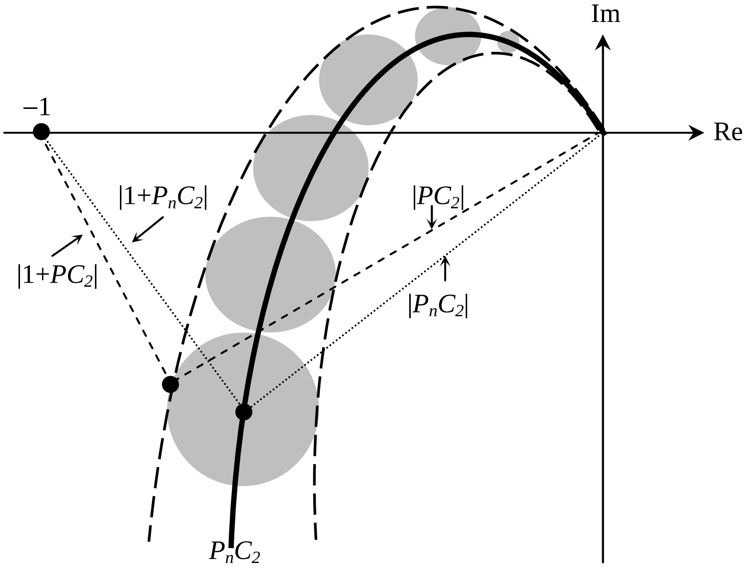

Figure 21. *Interpretation of the ratio in (2), which is the transfer function* Y/Y_n