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A MULTI AGENT PURSUIT-EVASION GAME MODEL FOR AIR ENGAGEMENT

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ABSTRACT

This paper considers the problem of pursuit evasion game model as an observer design problem. The agents are discrete linear subsystems operating in 3D space and the contribution of this paper is the formulation of the problem as an observer problem. Given the configuration matrix, one can simulate engagement of teams, deletion of agents and limited fuel, constrained input type control problems. The current paper explains how the system is set up.

INTRODUCTION

Pursuit and evasion problem has been studied by many researchers and fundamentally different perspectives have been stipulated. The problem has many aspects when there are multiple agents in a game. The availability of information to the agents, teams and missions make the problem even more challenging. In this paper, we consider a multi agent scenario where the agents are unit mass particles moving in 3D space. Every agent has three control inputs for x, y and z directions and every agent has a role in the game. The role is determined by a configuration matrix and we approach the problem as an observer design problem. The difficulty addressed here is the formulation of a single model whose role can change according to a change in the configuration matrix.

In the past, many researches have been done in the area of UAV control. Multi agent tasking is one of the research area. [Meng, He, Teo, Su and Xie, 2015] proposes a control logic and path optimization for cooperative multi agent search and tracking. [Hafez, Marasco, Givigi, Iskandarani, Yousefi and Rabbath, 2015] implements model predictive controllers for solving multi agent dynamic encirclement on a stationary target. The "extension-decomposition-aggregation (EDA)" is used for designing a decentralized multi agent formation control method in [Yang, Naeem and Fei, 2014] and a distributed formation control method which based on a differential game is studied in [Lin, 2014]. A circular formation method for multi agents with multiple leaders is studied in [Han, Dong, Yi, Tan, Li and Ren, 2016].

Several important contributions to the literature of pursuit-evasion games have been done. For a one-to-one engagement problem, a real time autonomous engagement method developed by using Approximate Dynamic Programming (ADP) in [McGrew, How, Bush, Williams and Roy, 2010]. One-to-one pursuit-evasion game in a three dimensional environment solved by a game theoretic approach in [Alexopoulos, Schmidt and Badreddin, 2014]. [He, Zu, Chang, Zhang and Gao, 2016] introduces an autonomuous maneuver decision metohod based on experiences of pilots in one-to-one engagement. An implementation of advantage function in a one-to-one UAV engagement based on relative geometry is studied in [Karli, Efe and Sever, 2015]. A multi model control framework is presented in [Üre and İnalhan,2012] for generating agile combat maneuvers autonomously. In a multiple pursuers against an evader scenario [Chen, Zha, Peng and Gu, 2016] considers a set of agents where one of them is evader and its mobility is superior than the pursuers'. The paper focuses on the conditions for capturing the evader. [Zhoua, Zhangb, Dingc, Huangd, Stipanoviće and Tomlin, 2016] approaches the problem of cooperative pursuit by minimizing the area of the generalized Voronoi partition of the

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evader. The setup of the problem contains multiple pursuers a single evader that is superior than the pursuers.

The multiple agent engagement against multiple agents is the most challenging problem. A framework for dynamic multi team antagonistic game is developed in [Zha, Chen and Peng, 2015]. In this paper, a tactical decision method proposed for a group of UAVs against antagonistic multiple ground targets based on perfect Bayesian Nash equilibrium. A cooperative air combat framework is proposed in [Duan, Wei and Dong, 2013] based on particle swarm optimization, ant colony optimization and game theory.

This paper is organized as follows: In the next section, the control design for multiple UAVs is introduced. “Configuration Matrix and The Pursuit-Evasion Game” section defines the multi agent pursuit-evasion game which depends on the configuration matrix. The problem defined and the simulation environment introduced in “Main Problem and The Generic System Model” section. Results of the simulations are shown in “Simulation Study” section. The last part includes the concluding remarks and future works.

PRELIMINARIES

Let a continuous time single dimensional dynamic system be $x=u$, where u is the input and x is the position. This model is discretized using forward Euler method with a sampling time denoted by T as follows. In the model given below, x_1 is the position, x_2 is the velocity

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_2(k) \\ x_2(k+1) &= x_2(k) + Tu(k) \\ y(k) &= x_1(k) \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}}_{A_d} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ T \end{bmatrix}}_{B_d} u(k) \quad y(k) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_d} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{1}$$

According to the above representation it should be noted that the pair (A_d,B_d) is controllable and the pair (C_d,A_d) is observable. An observer to this system can be given as in (2)

$$\hat{x}(k+1) = A_d\hat{x}(k) + B_d u(k) + L(y(k) - C_d\hat{x}(k)) \tag{2}$$

where $\hat{x}(k)$ is the state vector of the observer and L is the observer gain designed such that the eigenvalues of the matrix A_d-LC_d are strictly within the unit circle. This can be done easily via Bass-Gura or Ackermann formulas [Kailath, 1980]. We extend the agent dynamics to 3D and obtain the following discrete time dynamic system representation.

$$x(k+1) = \underbrace{\begin{bmatrix} A_d & 0 & 0 \\ 0 & A_d & 0 \\ 0 & 0 & A_d \end{bmatrix}}_{A_{3d}} x(k) + \underbrace{\begin{bmatrix} B_d & 0 & 0 \\ 0 & B_d & 0 \\ 0 & 0 & B_d \end{bmatrix}}_{B_{3d}} u(k), \quad y(k) = \underbrace{\begin{bmatrix} C_d & 0 & 0 \\ 0 & C_d & 0 \\ 0 & 0 & C_d \end{bmatrix}}_{C_{3d}} x(k) \tag{3}$$

The model above has the following state and input variables:

$$x(k) := \begin{bmatrix} p_x & v_x & p_y & v_y & p_z & v_z \end{bmatrix}^{\mathrm{T}} \quad u(k) := \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^{\mathrm{T}} \tag{4}$$

where p_x, p_y and p_z are positions in x, y and z directions and v_x, v_y and v_z are the velocities of the agent. Consider the agent in (3) is an evader. We design an observer to pursue it, i.e. the observer as given below.

$$\hat{x}(k+1) = A_{3d}\hat{x}(k) + B_{3d}u(k) + (y(k) - C_{3d}\hat{x}(k)) \otimes L \tag{5}$$

where L is as defined in (2) and $\otimes$ stands for the Kronecker tensor product. It is obvious that the agent in (4) follows the agent in (3) under the following assumptions, which hold true in the rest of this paper.

1. The agents have identical dynamical capabilities along all three Cartesian directions

2. Every agent is a unity mass player
3. Same observer gain is used for the sub-dynamics associated to each Cartesian direction
4. If an agent is following another agent, they know each other's input variable, i.e. $u(k)$.

CONFIGURATION MATRIX AND THE PURSUIT-EVASION GAME

Consider there are 2N agents. Clearly, a particular instant of time in the game can be described by a graph that describes the pursuers and evaders. Since there is a one-to-one engagement, we have an even number of nodes in the graph. In Figure 1, we see a 1-1 scenarios.

Figure 1: Left: Agent_1 follows Agent_2. Agent_1 is an observer.
Right: Agent_2 follows Agent_1. Agent_2 is an observer.

We define the configuration matrix as below.

$$Q_{ij} := \begin{cases} 1 & \text{if} \quad \text{Agent}_i \to \text{Agent}_j \\ 0 & \text{else} \end{cases} \tag{6}$$

According to above definition, we would have $Q = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for the left graph in Figure 1, and $Q = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ for the right graph in Figure 1. This definition of the configuration matrix puts forth several properties as listed below.

1. If there are $2N$ agents, Q is a matrix of dimensions $2N\times 2N$.
2. For a particular i and j, if Q_{ij}=1, then there cannot be another 1 in i-th row and j-th column of Q.
3. If $Q_{ij}=1$ we know that $Q_{ji}=0$.
4. The diagonal entries of Q are zero.

As a second example, we define a 4 agent game as illustrated in Figure 2, where 1 follows 2 and 3 follows 4, i.e. $Q_{12}=1$ and $Q_{34}=1$. All the remaining entries of Q are zeros. The corresponding Q matrix is given in (7)

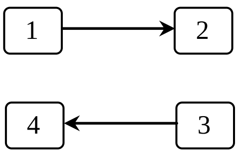

Figure 2: Engagement scenario for four agents

$$Q = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

In a dogfight, the advantage of an agent may change according to the relative position, and velocity vectors of one agent to another. There may be other parameters influencing the advantage of an agent and a mechanism to synthesize Q automatically could be designed. Yet, the simulation environment must address the switchings in the configuration. In an exemplar case, we may start with the regime in Figure 2, yet the game can switch to that in Figure 3 after a particular time. In such a case, the matrix Q is a time varying quantity.

Figure 4: Left: Engagement scenario for $t<t_{switch}$ Right: Engagement scenario for $t \geq t_{switch}$.

$$Q(t)=\begin{cases}\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix} & t<t_{switch}\\ \begin{bmatrix}0&0&0&1\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix} & t\geq t_{switch}\end{cases} \tag{8}$$

In above, two facts should be emphasized. 1 was following 2, but after switching 1 follows 4. So 1 is a follower before and after the switching. Considering 2, 1 was following 2 yet after the switching instant, 2 starts following 3, i.e. it changes mode and it becomes an observer. At this point, we need to distinguish two different types of agents in a game, namely, pursuers and evaders. Evaders are free signal generators and pursuers are the observers designed for them.

MAIN PROBLEM AND THE GENERIC SYSTEM MODEL

There are 2N agents involved in the pursuit-evasion problem. Every agent is a single player and there is no team behavior. The configuration matrix is time varying and the status of an agent may change from evader to pursuer or pursuer to evader. The problem is to develop a simulation environment that provides these properties.

Theorem: Let the subscript index i denote agent i. Let $F_i := \sum_{j=1}^{2N} Q_{ji}$, where F is a column vector. Let $r_i^c(k)$ be the reference signal for agent i, if agent i is an evader. The generic system model in (9) represents the pursuer and evader dynamics compactly and depending on the configuration matrix, the every subsystem's input is as defined in (10).

$$x_i(k+1)=A_{3d}x_i(k)+B_{3d}u_i(k)+F_i\sum_{j=1}^{N}\left(Q_{ij}\left(y_j(k)-C_{3d}x_i(k)\right)\otimes L\right) \tag{9}$$

$$u_i(k)=(1-F_i)\left(-K_ix_i(k)+r_i^c(k)\right)+F_i\sum_{j=1}^{N}Q_{ij}\left(-K_jx_j(k)+r_j^c(k)\right) \tag{10}$$

where K_i is the state feedback gain for the evader subsystem.

Proof by example: Consider the 4 agent game in Figure 2, where 1 follows 2, and 3 follows 4. For this game, $F = [1\ 0\ 1\ 0]^{\mathrm{T}}$. The equations above produce the following set of equations in this game.

$$x_1(k+1)=A_{3d}x_1(k)+B_{3d}u_1(k)+\sum_{j=1}^{N}\left(Q_{1j}\left(y_j(k)-C_{3d}x_1(k)\right)\otimes L\right) \tag{11}$$

$$u_1(k) = \sum_{j=1}^{N} Q_{1j}\left(-K_j x_j(k) + r_j^c(k)\right) \tag{12}$$

$$x_2(k+1) = A_{3d}x_2(k) + B_{3d}u_2(k) \tag{13}$$

$$u_2(k) = -K_2 x_2(k) + r_2^c(k) \tag{14}$$

$$x_3(k+1) = A_{3d}x_3(k) + B_{3d}u_3(k) + \sum_{j=1}^{N}\left(Q_{3j}\left(y_j(k) - C_{3d}x_3(k)\right)\otimes L\right) \tag{15}$$

$$u_3(k) = \sum_{j=1}^{N} Q_{3j}\left(-K_j x_j(k) + r_j^c(k)\right) \tag{16}$$

$$x_4(k+1) = A_{3d}x_4(k) + B_{3d}u_4(k) \tag{17}$$

$$u_4(k) = -K_4 x_4(k) + r_4^c(k) \tag{18}$$

Using the corresponding configuration matrix (Q), the above set of equations can be rewritten as below.

$$x_1(k+1) = A_{3d}x_1(k) + B_{3d}u_1(k) + \left(y_2(k) - C_{3d}x_1(k)\right)\otimes L \tag{19}$$

$$u_1(k) = -K_2 x_2(k) + r_2^c(k) \tag{20}$$

$$x_2(k+1) = A_{3d}x_2(k) + B_{3d}u_2(k) \tag{21}$$

$$u_2(k) = -K_2 x_2(k) + r_2^c(k) = u_1(k) \tag{22}$$

$$x_3(k+1) = A_{3d}x_3(k) + B_{3d}u_3(k) + \left(y_4(k) - C_{3d}x_3(k)\right)\otimes L \tag{23}$$

$$u_3(k) = -K_4 x_4(k) + r_4^c(k) \tag{24}$$

$$x_4(k+1) = A_{3d}x_4(k) + B_{3d}u_4(k) \tag{25}$$

$$u_4(k) = -K_4 x_4(k) + r_4^c(k) = u_3(k) \tag{26}$$

This example shows that the equation pair in (9)-(10) produce the pursuer and evader dynamics correctly, and it can be inferred from this result that a possible change in the agent configuration can properly be handled. In the next section, we consider a simulation work to demonstrate that the game is played correctly.

SIMULATION STUDY

In the simulations, we consider 6 agents and the following configuration matrix, which undergoes two changes during the course of the simulation.

$$Q := \begin{cases} Q^1 & 0 \le k < 90 \\ Q^2 & 90 \le k < 180 \\ Q^3 & k \ge 180 \end{cases} \tag{27}$$

where k is the discrete time index and

$$Q^1_{6\times 6} := \begin{cases} Q^1_{ij} = 1 & ij \in \{12, 34, 56\} \\ Q^1_{ij} = 0 & \text{Other} \end{cases} \tag{28}$$

$$Q_{6\times6}^{2} := \begin{cases} Q_{ij}^{2} = 1 & ij \in \{14, 36, 52\} \\ Q_{ij}^{2} = 0 & \text{Other} \end{cases} \tag{29}$$

$$Q_{6\times6}^{3} := \begin{cases} Q_{ij}^{3} = 1 & ij \in \{23, 45, 61\} \\ Q_{ij}^{3} = 0 & \text{Other} \end{cases} \tag{30}$$

According to the above definition, we can deduce the following table of roles for each agent. In the table, we see that an agent follows another agent for the first period, yet in the second period it changes its target and follows another agent. The same agent changes its role in the third column and becomes an evader. With such a strategy, we can see every possible combination that can occur in an exemplar engagement situation.

Table 1: Roles of the Agents as Time Passes

	$0 \le k < 90$	$90 \le k < 180$	$k \ge 180$
Agent_1	Pursuer	Pursuer	Evader
Agent_2	Evader	Evader	Pursuer
Agent_3	Pursuer	Pursuer	Evader
Agent_4	Evader	Evader	Pursuer
Agent_5	Pursuer	Pursuer	Evader
Agent_6	Evader	Evader	Pursuer

During the simulations, we choose the initial condition in (31) and reference signals for evaders as in (32). The remaining settings are tabulated in Table 2.

$$[x_1(0) \quad x_2(0) \quad x_3(0) \quad x_4(0) \quad x_5(0) \quad x_6(0)] = \begin{bmatrix} -1 & -1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{31}$$

$$[r_1(k) \quad r_2(k) \quad r_3(k) \quad r_4(k) \quad r_5(k) \quad r_6(k)] = (k+1)\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

Table 2: Simulation Settings

Eigenvalues of A_d-LC_d	[0.95 0.85]
Eigenvalues of A_{3d}-$B_{3d}K$	[0.90 0.80 0.90 0.80 0.90 0.80]
Simulation step size (T)	0.01 sec.
Final Time	27 sec. (k_{max}=270)

In Figure 3, we illustrate the three regimes of engagement in different subplots. The 3D illustrations of motion show that an agent freely moves and another catches it till a configuration change occurs. The behavior is successfully maintained in the second regime and the third regime, where roles change dramatically.

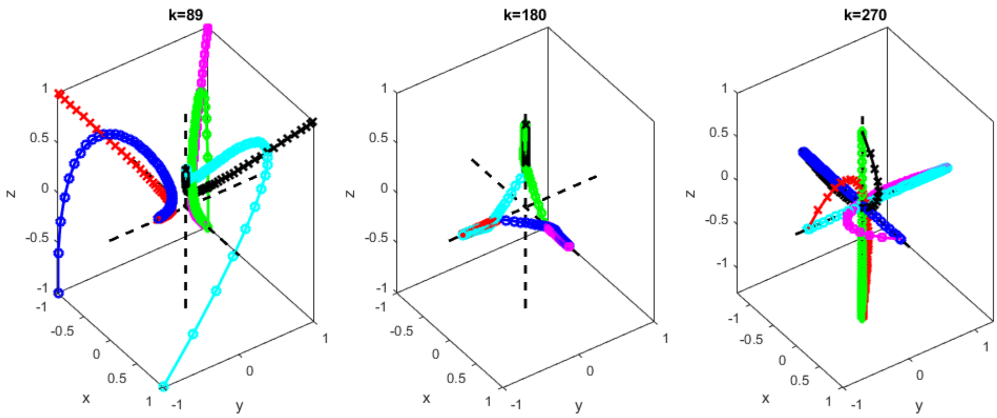

Figure 3: Left: The motion of the agents for Q_1; Middle: Motion for Q_2; Right: Motion for Q_3

CONCLUSION

Air engagement is a complex problem that displays a number of difficulties. The agent based maneuver decisions differ significantly from team based motion maneuver decisions. Heterogeneity of the agents is another dimension making the problem tedious. In a general context of pursuit-evasion game, the geometric and physical constraints also need to be addressed. In this paper, we consider a homogeneous group of agents that engage one-to-one as described by a configuration matrix. The matrix is a time varying one addressing a change in the game. The goal here is not to design the game, instead, we focus on setting up the pursuit-evasion problem as an observer design problem and assume the availability of the positions of the agents. The simulations show an exemplar case with two switchings and we see that the pursuer attached to a particular evader approaches its target very quickly. Our future goal is to handle the problem settings displaying the aforementioned difficulties.

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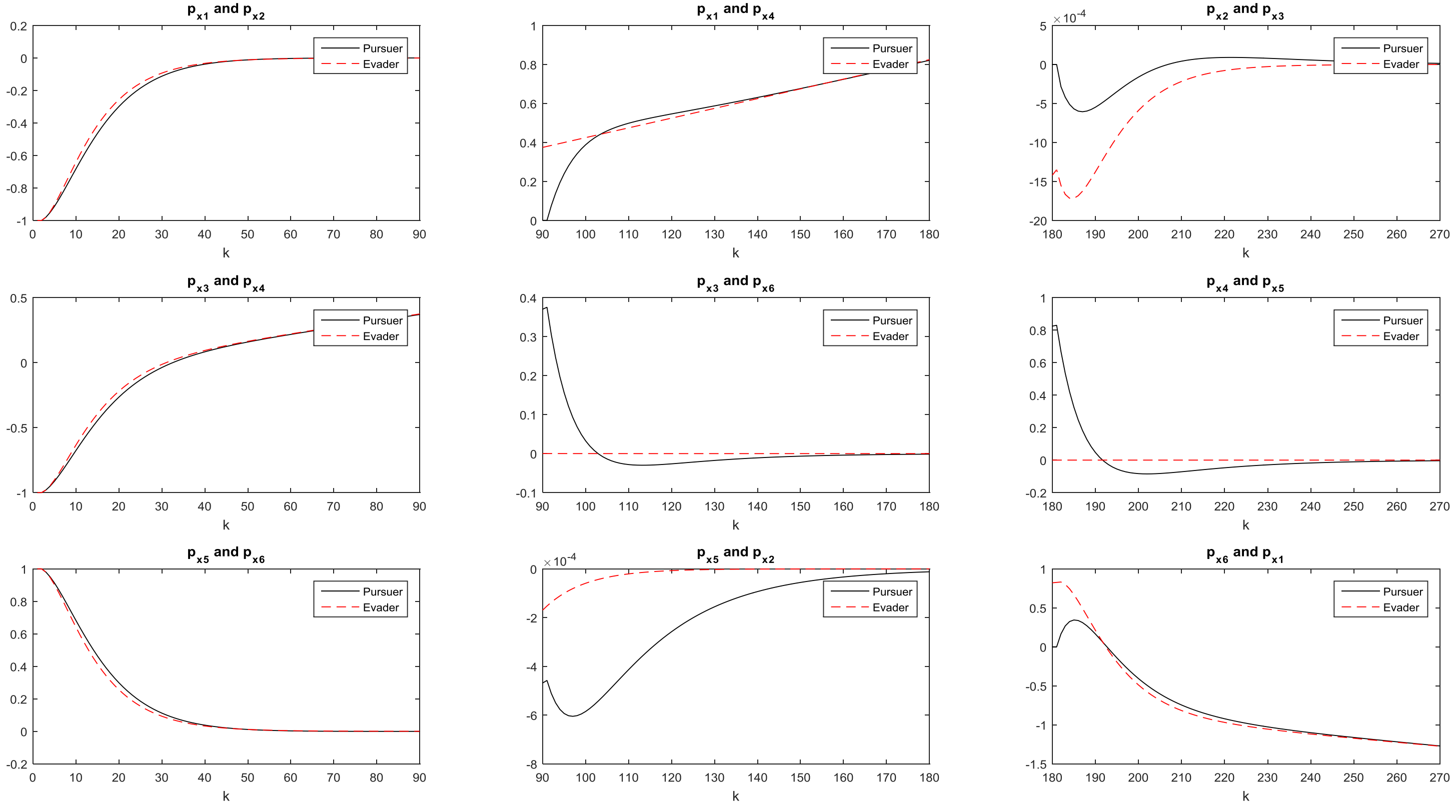

Figure 4: x axis measurements for all six agents.

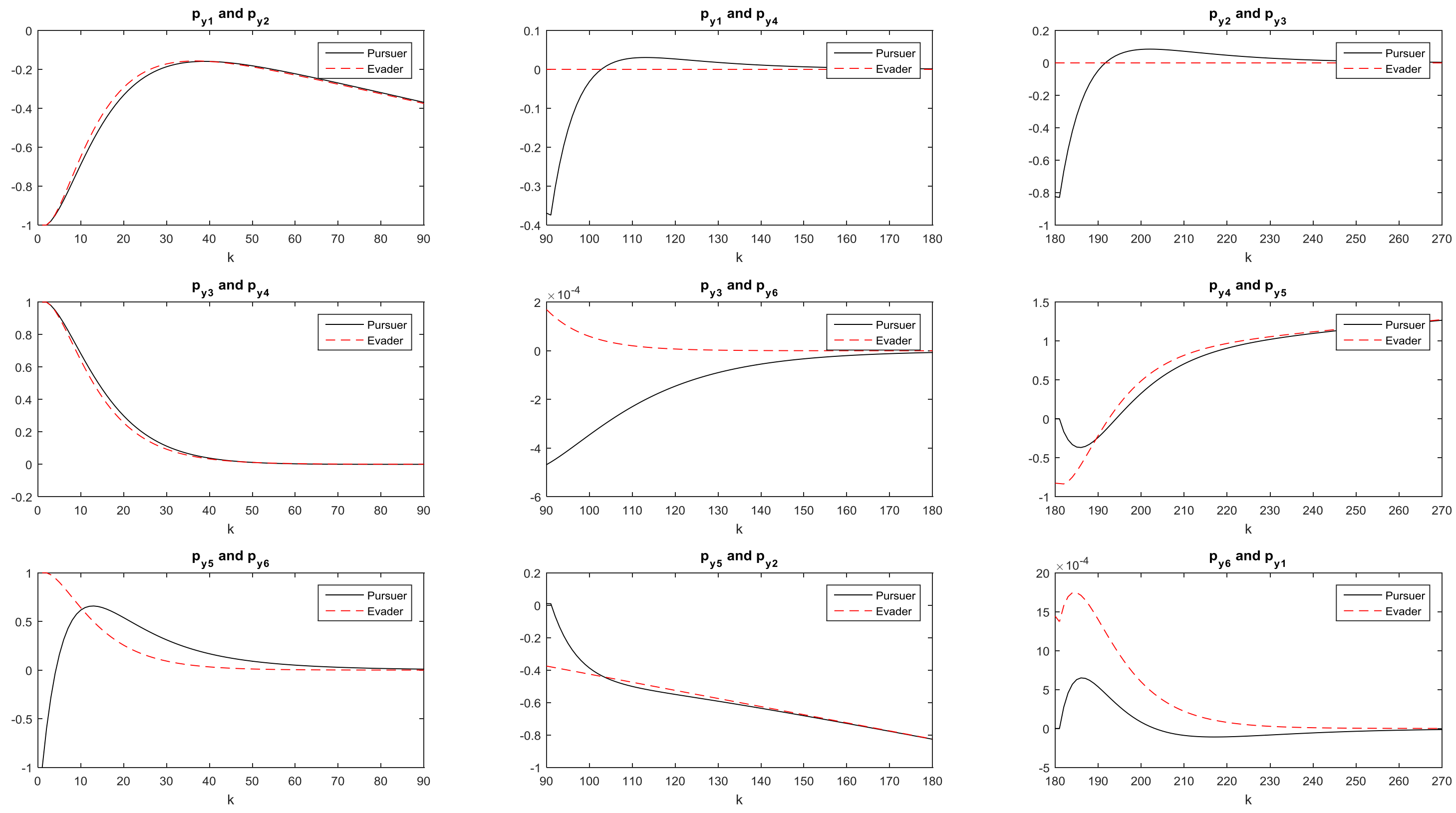

Figure 5: y axis measurements for all six agents.

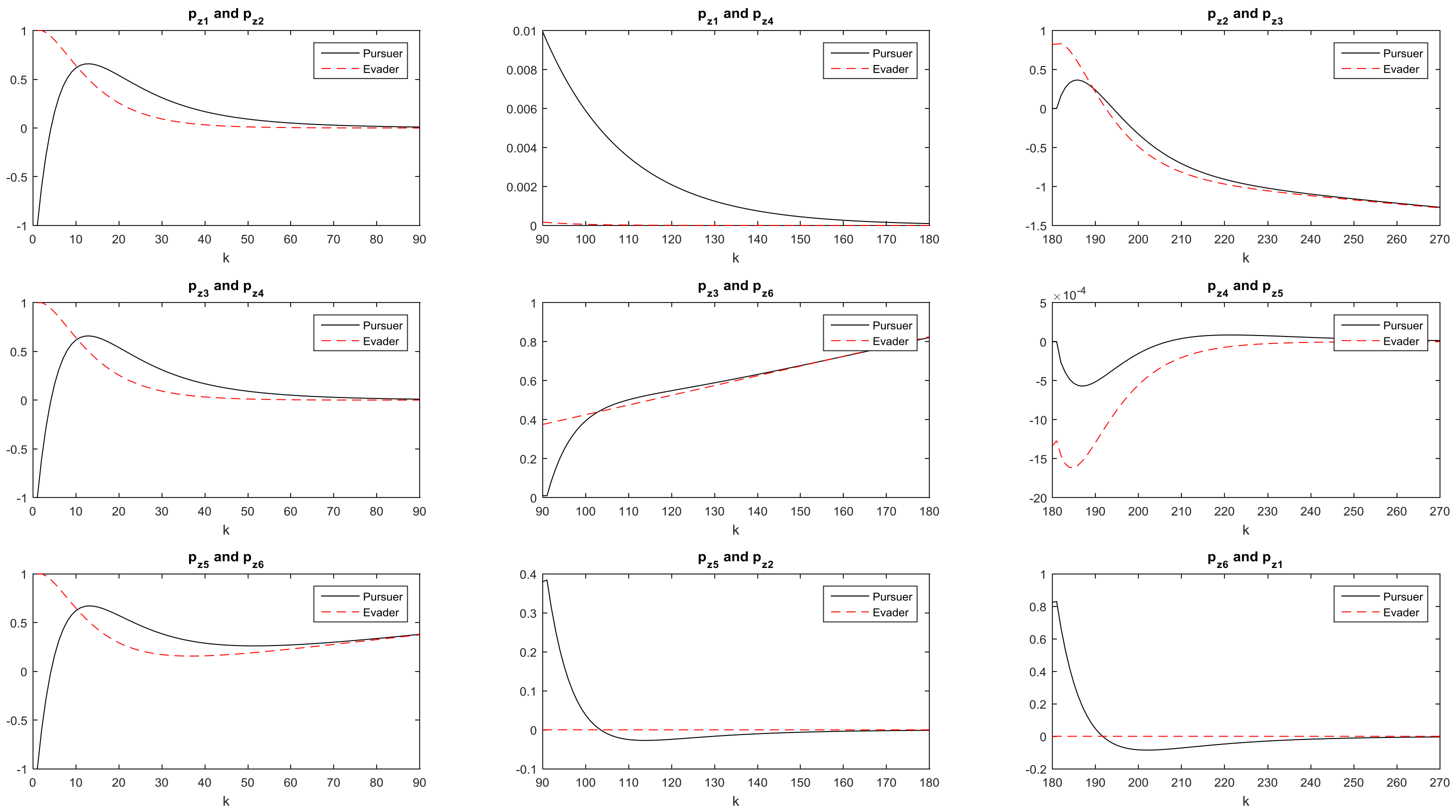

Figure 6: z axis measurements for all six agents.

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