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A GAME THEORETIC MODEL FOR MULTI AGENT PURSUIT-EVASION PROBLEM

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ABSTRACT

In this paper a method proposed for multi UCAV air engagement problem in four steps. First the superiorities calculated for each opponent pair in 3d space. Then possible strategies determined and a prediction calculated for each agent TTI (Time to Intercept) in each strategy. In third step Nash equilibrium implemented to calculated strategies for selecting best strategy for each team. In the last step an observer designed for pursuing selected opponent act upon proposed strategy by game theory. The contribution of this study to literature is combining the tactical level decision and operative level air engagement. Method can be implemented to 2 by 2, 3 by 3 and 4 by 4 teams. 3 by 3 air engagement simulation results are presented.

INTRODUCTION

UCAVs (Unmanned Combat Aerial Vehicles) are becoming key figures in battle field not only for reconnaissance and surveillance but also as a weapon for destroying targets and engaging opponents at air combat field. Autonomy in this area is a very challenging problem because of the fast and unlimited nature of air engagement. However there is not enough autonomy in reality there are many researches for autonomous air engagement. There many valuable contributions in literature. The literature reviewed in four main sections.

Single UAV control **–** agile combat maneuvers are generating autonomously by a multi model control framework designed in [Üre and İnalhan, 2012].

One-to-One UAV engagement - Approximate Dynamic Programming (ADP) is implemented to one-to-one air engagement in [McGrew, How, Bush, Williams and Roy, 2010]. Game theory is used in [Alexopoulos, Schmidt and Badreddin, 2014] for making decision for single pursuer and single evader. [Karli, Efe and Sever, 2015] is introduced an advantage function and use this function for creating predefined agile BFMs (Basic Fighter Maneuvers) [Shaw, 1985] for evader. The experiences of pilots are used in [He, Zu, Chang, Zhang and Gao, 2016] for making decisions in pursued-evasion game.

Multi UAV tasking – a differential game based method is proposed in [Lin, 2014] for decentralized formation control and also “Extension-Decomposition-Aggregation (EDA)” method is implemented for same problem in [Yang, Naeem and Fei, 2014]. A control logic is introduced by [Meng, He, Teo, Su and Xie, 2015], for optimizing paths in search and tracking tasks undertaken by multi UAVs in a cooperation. Model predictive controllers are designed by [Hafez, Marasco, Givigi, Iskandarani, Yousefi and Rabbath, 2015] for multi UAV dynamic encirclement problem and [Han, Dong, Yi, Tan, Li and Ren, 2016] is introduced a method for circular formation, controlled by multiple leaders.

Multi UAV engagement – The most challenging problem is multi UCAV air engagement to multiple dynamic opponents in 3d space. Particle swarm optimization based framework is developed by [Duan, Wei and Dong, 2013] for the cooperative air engagement problem. Game theoretic approach is used in [Zha, Chen and Peng, 2015] for solving the problem of multi UCAVs against antagonistic ground agents. A tactical target assignment decision method is proposed for antagonistic teams with multiple

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UCAVs in [Özpala, Efe and Sever, 2017]. UCAVs select the best assignment for the advantage of their teams by game theoretic approach.

This paper is organized as follows: In the next section, the problem about multi UCAV engagement is defined and the proposed method for this problem is introduced. “The Model for Multi Agent Pursuit-Evasion” section defines the proposed method in detail. 3-by-3 antagonistic teams simulation and the results represented in “Simulation Example” section. “Conclusion” section includes the concluding remarks and future works.

PROBLEM DEFINITION AND THE PROPOSED METHOD

Even the autonomous control of unmanned aircraft is a difficult problem; air combat and also multiple air combat are very challenging problems. The main problem is the control of air vehicles as a pursuer in a 3 dimensional space. For multi UAV air combat, selecting the right match of antagonistic agents and their role is also a big problem. The multi UAV engagement problem is different from multi manned aircraft engagement, because there is no necessity to protect the pilot's life. The only goal is keeping alive more agent than your opponent.

Problem defined as a two player zero sum game with incomplete information and Nash Equilibrium searching for both teams. There are two antagonistic teams which are blue team and red team. Each team has multiple agents which are placed in 3D space. Each agent knows the spatial situation and behavior of its teammates and the spatial situations of opponents. In this scenario, it is not easy to matching opponents in an effective way. There are 4 main steps. The process flow of proposed method is shown in Figure 1.

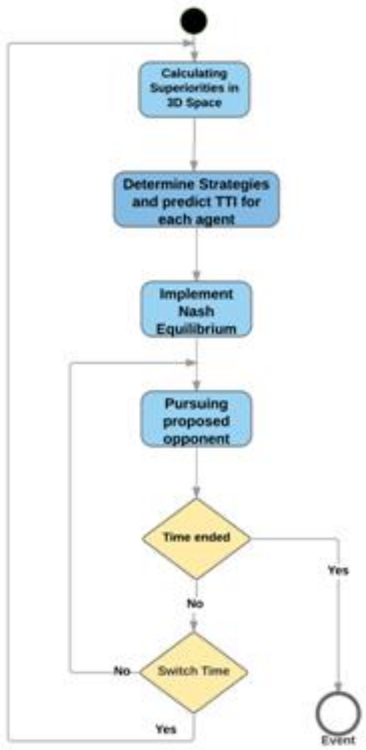

Figure 1- Process Flow

THE MODEL FOR MULTI AGENT PURSUIT-EVASION

In this section the propose method for multi UCAV engagement is explained in detail. This section organized as the steps in the Process Flow diagram.

Calculating Superiorities

Superiorities calculating for each agent pair depend on their relative geometry and velocities. Total superiority is the weighted sum of 3 sub superiority. 3 sub superiorities and the weighted sum are;

<u>Angle Superiority:</u> The representation of one-to-one engagement in a 3d space shown in Figure 2. φ and q are ATAs (Antenna Terrain Angle) for blue and red UAV respectively. d is the distance between two UAVs.

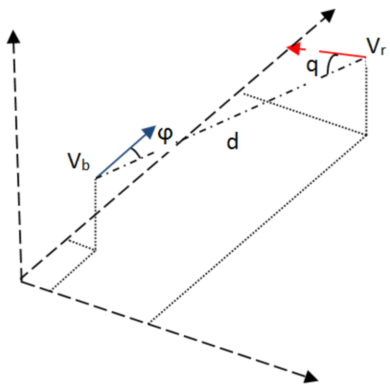

Figure 2 Angle Superiority

The Formula for angle superiority is;

$$S_a = \frac{|q| - |\varphi|}{180} \tag{1}$$

Range Superiority: The superiority of range depends on the distance of UAVs and the maximum and minimum range of their loaded weapons. The definition of variables is;

$$R = (W_{R_{max}} + W_{R_{min}})/2, \quad \sigma = 2 \times (W_{R_{max}} - W_{R_{min}}) \tag{2}$$

And the formula for range superiority is;

$$S_r = e^{-\left(\frac{R-R_0}{\sigma}\right)^2} \tag{3}$$

Speed Superiority: The speed superiority depends on the absolute velocity of each UAV. The formula for speed superiority is;

$$S_s = 1/(1 + e^{-6(\frac{V_r}{V_b})+6}) \tag{4}$$

Total Superiority: Total superiority is the weighted sum of three sub superiorities. The formula for speed superiority is;

$$S_T = k_1 S_a + k_2 S_r + k_3 S_s$$
$$0 < k_1, k_2, k_3 < 1 \ and \ k_1 + k_2 + k_3 = 1 \tag{5}$$

After calculating superiorities for UAV pairs $supMat_{n \times n}$ obtained. Each UAV pair get a value between $1 \ and \ -1$. Positive values indicate blue UAV is superior and negative values indicate red UAV is superior. 0 means there no superiority between these UAVs.

Determining Strategies and Predicting TTI

There are two antagonistic teams and each team has n UAV. Even though each UAV can engage only one opponent, team members engaged to same opponent as offensive or defensive manner. In this case there are n^n possible matching for each team and there are $n^n \times n^n$ different strategy for game. In fact some of these strategies are senseless but it is not obvious which strategies are senseless before implementing game theory.
By using strategy matrix, agent opponent matrix created which needed by TTI prediction algorithm. Agent opponent matrix which shown below is represents all agents in rows and their opponents in columns.

$$aoMat_{2n \times n} = [supMat \ \ -supMat'] \tag{6}$$

The algorithm for TTI prediction is shown in Algorithm 1.

```
call CalcStrategies(k = 2n, aoMat)
function CalcStrategies()
        for (i = 1 to n)  (loop by count of opponent relatively for each agent)
        indAgent = k (index of agent)
        indOpponent = i (index of Opponent)
        line(indAgent) =  aoMat (indAgent, indEnemy) (superiority assigning)
        if (indAgent  = 2n)
                for s = 1 to 2n
                if(line(s)>0)( if offensive. If not it isn't change the enemy's life time)
                        if(opponent is defensive)
                                TTI(indEnemy) = (1 − line(s)) × defCo
                        Else
                                TTI(indEnemy) = (1 − line(s)) × offCo
        If( k>1) call CalcStrategies(k + 1) (Recall function recursively)
```

Algorithm 1 TTI Prediction Algorithm

By this algorithm, strategy matrix is created which includes each UAVs TTI for all strategy pairs.

Implement Nash Equilibrium

Game theory is a decision method for competitive and collaborative decision makers. The latest practices are based on articles published by John Forbes Nash between 1950-1953 [Nash, 1950 and Nash, 1951]. Game theory primarily used in economics and mathematics then in politics, social sciences and biology. From 1900s game theory implemented to engineering fields.

For implementing game theory a game matrix is needed. By strategy matrix, each UAV's life time is obtained and also it is possible to find the remaining UAVs for both teams in each strategy. The rate of total UAVs in a team to difference between remaining UAVs is the gain for related strategy.

$$Gain = (remaing\ Blue\ UAV - remaing\ Red\ UAVs)/n$$
$$-1 \leq Gain \leq 1 \tag{7}$$

Then we obtain a zero sum game matrix $gtMat_{n^n \times n^n}$ by Algorithm 2. If an UAV not engaged by any of its opponents, infinite TTI is attended.

```
For i = 1 to n^n
    For j = 1 to n^n
        indStraMat = n^n × (i − 1) + j
        Sort agents ascending by TTI in stratgyMat(indStraMat, :)
        For j = 1 to 2n
            If (stratgyMat(indStraMat, n) <> ∞ and opponent not defeated)
             Eliminate agent
        End For
        Calculate Gain
        gtMat(i, j) = Gain
    End For
End For
```

Algorithm 2 Creating Game Matrix

Game Theory implement in three steps.

- In first step "Pure NEs" are searching. If there is a pure NE this means there is a strategy pair which is always better for each team then other strategies. This strategy is strictly chosen by teams.
- If there is no Pure NE, dominated strategies are searching and eliminating. A team's dominated strategies are always worst than its opponent's strategies. This makes these strategies never chosen by relevant team.
- In last step "Mixed Nash Equilibrium" is calculating. By Mixed NE calculation, probability coefficients are finding for significant strategies. Mixed NE is calculating by linear programming. This process is a computationally expensive problem. Before calculating Mixed NE, a reduction method performed to game matrix [Özpala, Efe and Sever, 2017].

By the previous sections the significant strategies obtained with their probabilities. By this information an engagement decision matrix can be created. Engagement decision matrix $Q_{2n\times 2n}$ is using in role assignment for each UAV. Rows and columns are formed by all UAVs. The intersections of opponents are get values {-1, 0, 1} in Q matrix. Negative 1 means that the UAV at row, engaged defensively to the UAV at the intersected column.

0 means there is no engagement between relevant UAV pair. Positive 1 means that the UAV at row, engaged defensively to the UAV at the intersected column. An example Q matrix shown below for 3 by 3 engagement.

Table 1 Q Matrix For 3 by 3 Engagement

	B1	B2	B3	R1	R2	R3
B1	0	0	0	0	0	1
B2	0	0	0	0	1	0
B3	0	0	0	0	-1	0
R1	1	0	0	0	0	0
R2	0	0	1	0	0	0
R3	0	0	-1	0	0	0

Pursuing opponent as an observer:

The visualization of the game is achieved through defining a set of observer structures. The agents move in 3D space and each has a unit mass. The accelerations is therefore due to the applied input that is to be designed. During the evolution of the game, for example, an agent, say agent i, is following agent j during a time period, say $t1 \leq t \leq t2$. The model in this case is set up as defining ith agent as an observer and jth agent as the system being observed. Considering the change of game scenarios, an agent may change its role from pursuer (observer) to evader (escaper). The configuration of a frozen time is defined by a matrix, the ijth entry is unity if agent i follows agent j, and zero otherwise. The synthesis of configuration matrix is performed via the approach presented in this paper.

SIMULATION EXAMPLE

A generic frame work is designed for simulation. In this example the simulation is running by;

Number of UAVs in each team = 3,

Final Time for simulation= 120,

Switch count = 2.

The initial positions are;

Table 2 Initial Positions

	B1	B2	B3	R1	R2	R3
X	0.1	0.5	0.9	0.2	0.4	0.9
Y	0.2	0.1	0.25	0.8	0.7	0.75
Z	0.8	0.6	0.65	0.7	0.9	0.75

Calculated superiority matrixes shown in Table 3.

Table 3 Superiority Matrixes

First Switch

	R1	R2	R3
B1	-0.36	-0.36	0.50
B2	0.36	0.35	0.62
B3	-0.30	-0.35	0.58

Second Switch

	R1	R2	R3
B1	0.37	0.45	0.84
B2	-0.50	0.30	0.54
B3	-0.58	-0.68	0.31

The decision made by game theoretic approach shown in Table 4.

Table 4 Q Matrixes

First Switch

	B1	B2	B3	R1	R2	R3
B1	0	0	0	0	0	1
B2	0	0	0	0	1	0
B3	0	0	0	0	-1	0
R1	1	0	0	0	0	0
R2	0	0	1	0	0	0
R3	0	0	-1	0	0	0

Second Switch

	B1	B2	B3	R1	R2	R3
B1	0	0	0	0	1	0
B2	0	0	0	0	0	1
B3	0	0	0	0	-1	0
R1	0	1	0	0	0	0
R2	0	0	1	0	0	0
R3	0	-1	0	0	0	0

The engagement results for first switch shown in Figure 3 and the second switch is shown in Figure 4. At the title of the graphics, Pos-1, Pos-2 and Pos-3 indicates X, Y and Z positions respectively.

P-2→E-4 means; second agent is pursuer and forth agent is evader. The agents between 1-3 are Blue team's members and agents between 4-6 are the red team's members. k is the time variant in seconds.

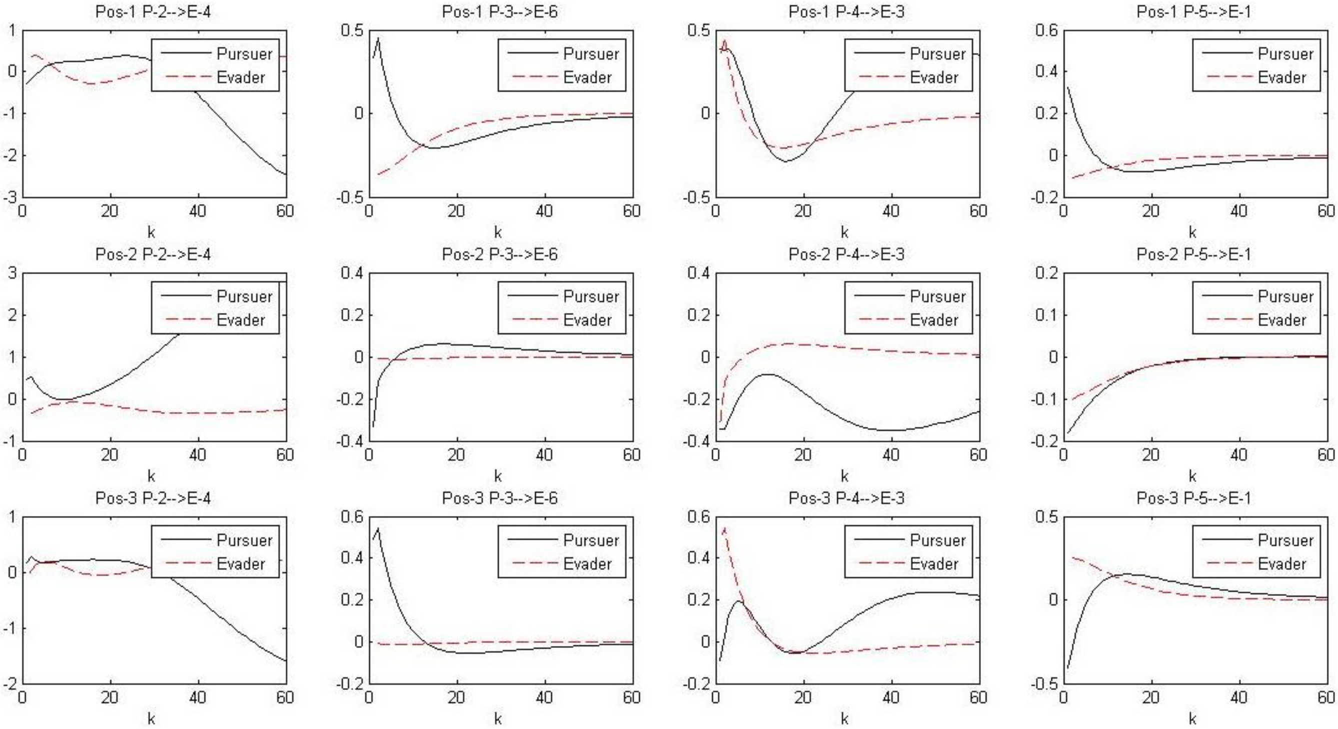

Figure 3 First Switch (T=0-60)

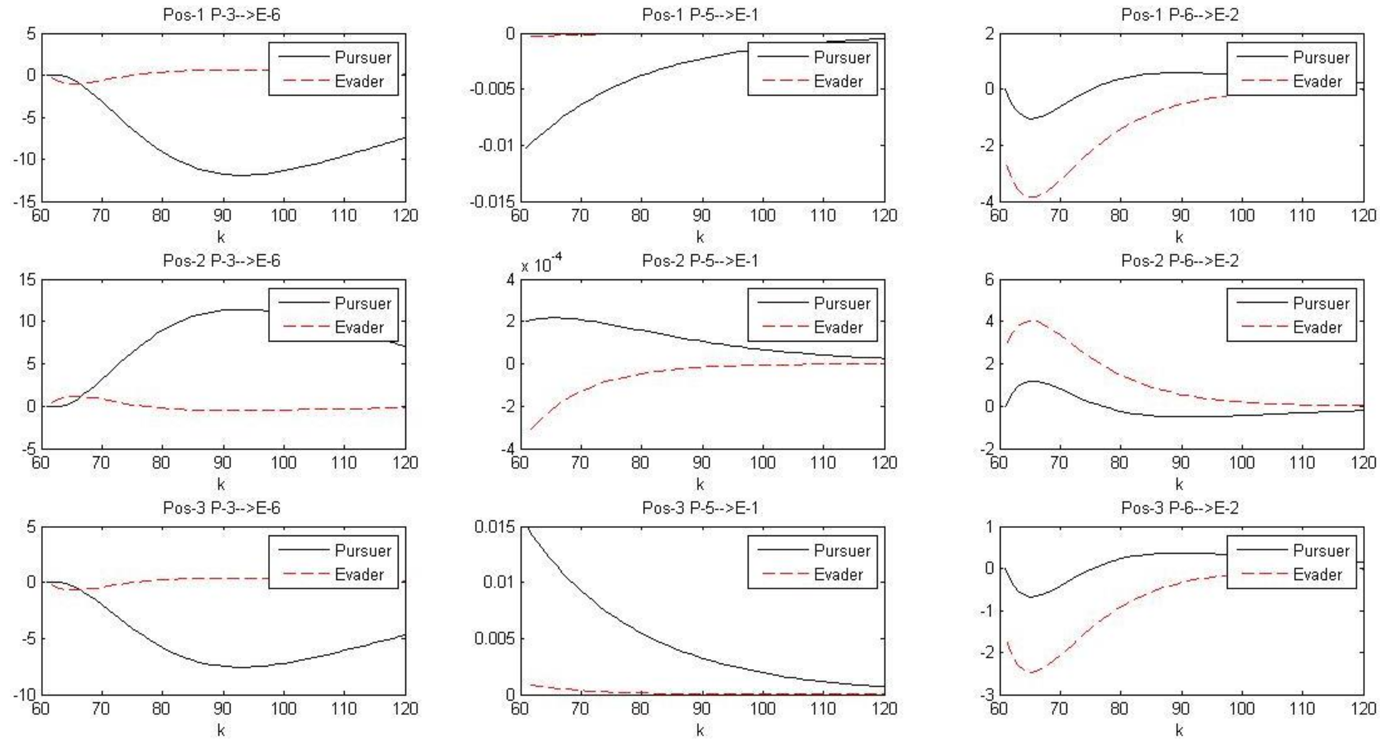

Figure 4 Second Switch (T=61-120)

CONCLUSION

Multi UCAV engagement is the most challenging problem in the area of UAV autonomy. In this study a method proposed for combining the tactical level multiple dynamic target assignment decision and operative level engagement behavior in 3d space. The UCAVs make the best decision for their own teams and also engaged to assigned opponent autonomously. The results shows that the decisions are successful and the agents are enhanced their advantage by approaching as an observer. The future works are enhancing the performance of decision method and controlling UCAVs by the 6DOF forces.

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