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COMPARISON OF THREE-DIMENSIONAL GUIDANCE LAWS FOR MISSILE GUIDANCE SYSTEM

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ABSTRACT

This paper presents comparisons of the behaviors of three-dimensional guidance laws for two different target movements. These guidance laws are the Proportional Navigation (PN) guidance law, the Augmented Proportional Navigation (APN) guidance law, the Proportional-Integral-Derivative (PID) guidance law as well as the Sliding Mode (SM) guidance law. Firstly, missile and target are modeled in three-dimensional engagement geometry. Secondly, the designed guidance laws are evaluated against two different target model: the non-maneuvering target and maneuvering target. Also, this evaluation is made in the presence of external disturbances. The primary purpose of a guidance law is to reduce miss distance and in accordance with this purpose, the navigation constant used in the classical guidance laws has been determined properly. Furthermore, the saturation function is used to eliminate the chattering phenomenon, which is the most important disadvantage of the presented SM guidance law. Finally, these numerical simulation results are obviously revealed that the presented guidance laws have an ideal target intercepting ability and are more robust to external disturbance.

INTRODUCTION

Missile guidance laws have been one of the popular research areas for decades because results of these laws play a critical role in determining the missile's performance. The Proportional navigation (PN) is a well-known classical guidance strategy because of its simple structure, being easy to implement and having high efficiency [Nesline & Zarchan, 1981; Zarchan, 2012] . Studies on the PN guidance law have been carried out since the first day of the missile guidance research and therefore this law serves as the basis for comparing the performances of other proposed guidance laws. Several modifications of the PN guidance law have been suggested to improve performance. Those are pure proportional navigation (PPN), true proportional navigation (TPN), generalized TPN (GTPN) and augmented proportional navigation (APN) guidance laws. While TPN and PPN are categorized according to the direction of the acceleration commands, APN is calculated by considering the target acceleration. Still, PN guidance law and its variants have some major drawbacks. The difficulties of applying these guidance laws have become prominent in the presence of large target maneuvers as well as uncertainty conditions such as parameter

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variations and disturbances. To overcome those difficulties and enhance the performance, numerous classical and modern missile guidance laws have been proposed by using control methods and artificial intelligence [Golestani, M. Mohammadzaman, 2015; Lin, 2003; Rajasekhar, 2000; Yamasaki, Balakrishnan, Takano, & Yamaguchi, 2013).

The Proportional-Integral-Derivative (PID) control method is most known classical control method and is preferred thanks to its nature of being simple and easy to apply in every area. This law depends not only on the LOS rate but also on the LOS angle in order to overcome disturbances and miss distance problems. Evcimen et al. [Evcimen & Leblebicioğlu, 2013] developed an adaptive, optimal proportional-integral guidance law by using PID control method to increase efficiency against fast maneuvering target. PID parameters are determined as adaptive by using linear optimal control theory. Golestani et al. [Golestani, M. Mohammadzaman, 2015] proposed a state-of-art guidance algorithm, which developed by using PID guidance law. In this paper, a circle criterion has been employed for the stability analysis. Results showed that it can prominently make better the guidance performance for a maneuvering target according to classical PN guidance law.

Sliding mode control (SMC) is a robust and nonlinear control method in the presence of uncertainty of the system parameters as well as external disturbances and it also has been used as control method in many studies for many years. Up to now, sliding mode control has been compared against several guidance laws by many researchers for designing missile guidance systems. SMC has provided better results compared to the others due to having many advantages such as robustness against uncertainties, parameter variations as well as disturbances. Moon et al. [Moon, Kim, & Kim, 2001] demonstrated a guidance law based on the variable structure control by taking the target acceleration bound during maneuvering into account. Shtessel et al. [Shtessel, Shkolnikov, & Levant, 2007] presented the smooth second-order sliding mode (SSOSM) guidance strategy and compared it to the PN guidance law. In 2010, Lee et al. [C. Lee, Kim, Tahk, & Kim, 2010] conducted research on guidance law by using the SMC method for two different movements of the target. Zhou et. al. [Zhou, Song, Song, & Niu, 2014] developed two guidance laws in two-dimensional engagement by using the fast sliding mode guidance law (FSMG) and the variable dynamic sliding mode guidance (VDSMG) law. Results of this paper stated that FSMG is better than VDSMG in terms of convergence time and at eliminating the chattering phenomenon. Lee and Kim [Lee & Kim, 2013] proposed guidance law with sliding mode by using dual sliding surfaces. Also, it should be mentioned that SMC has a significant disadvantage, which is called as chattering phenomena. To eliminate or at least reduce this phenomenon, different methods are proposed, such as boundary layer, saturation function, sigmoid functions, second-order SMC [Shtessel et al., 2007; Song, Song, & Zhou, 2016].

In practice, the target-missile relative motion happens in a three-dimensional environment. The mathematical model of this environment comes from second-order nonlinear and coupling differential equations. There are many studies exist in which the guidance laws are designed in two planar geometries [Guo, Li, & Zhou, 2019; Moosapour, Alizadeh, & Khanmohammadi, 2013; Biswas, Kumar, & Maity, 2018]. In this way, the cross-coupling effects are ignored, and the design and analysis of the missile-target relative motion are very easily made thanks to these simplifications and some assumptions. However, if it is expected to get closer results to the real system, guidance law should be designed by taking the 3D engagement geometry into account. At present, many three-dimensional guidance laws have been studied by many researchers [Yang & Yang, 1995; Moran & Altılar, 2005; Kumar & Ghose, 2014; Wang, Jin, & Li, 2018; Lei & Li, 2012]. Yang and Yang [Yang & Yang, 1995] presented TPN guidance law; the generalized and the realistic TPN guidance law are described and solved in three-dimensional geometry. Also, Moran et. al. [Moran & Altılar, 2005] studied on TPN guidance law in 3D geometry. Kumar and Ghose [Kumar & Ghose, 2014] designed SMC guidance law by considering impact angle constraints in 3D geometry. In order to improve robustness of guidance system, Wang et al. [Wang, Jin, & Li, 2018] proposed a three-dimensional adaptive dynamic surface guidance which involved the second order dynamic of missile autopilot as well as acceleration constraints. Lei et al. [Lei & Li, 2012] used a nonlinear backstepping control approach for maneuvering targets.

The main contribution of this paper is to compare results of different guidance laws for 3D engagement geometry. PN guidance law, APN guidance law, Proportional-Integral guidance law, SMC based guidance law are designed for missile-target engagement in the presence of external disturbances in 3D geometry. 6-DOF numerical simulation is performed to confirm the effectiveness of the proposed guidance laws.

This paper is organized as follows. The guidance mathematical model is briefly presented in section 2. The following section has the design details of the guidance laws. PN and APN guidance laws are explained, after then PI and SMC guidance laws are designed in section 3. Then in section 4, simulation results and comparison of performance guidance laws are reported considering different scenarios. Section 5 draws the conclusions.

THE GUIDANCE MATHEMATICAL MODEL

The missile-target relative motion in three-dimensional environment is briefly explained in this section and is presented in Figure 1.

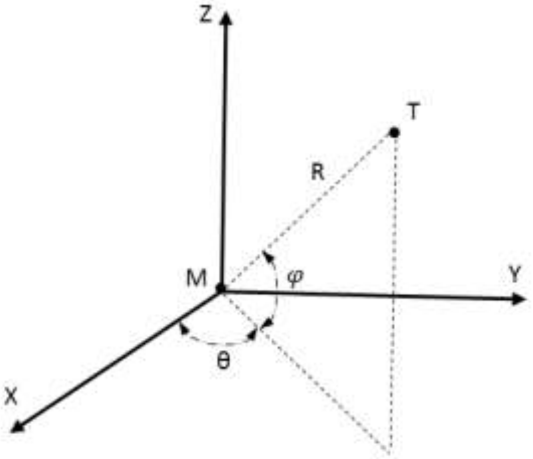

Figure 1. Three-dimensional engagement geometry

T is the target, M is the missile and R is the range between target and missile. θ and φ denote the elevation and the azimuth angles of the line-of-sight (LOS). The following second-order nonlinear differential equations can pressed by

$$\ddot{R} - R\dot{\varphi}^2 - R\dot{\theta}^2 cos^2\varphi = a_{TR} - a_{MR} \tag{1}$$

$$R\ddot{\theta}cos\varphi + 2\dot{R}\dot{\theta}cos\varphi - 2R\dot{\theta}\dot{\varphi}sin\varphi = a_{T\theta} - a_{M\theta} \tag{2}$$

$$R\ddot{\theta} + 2\dot{R}\dot{\varphi} + R\dot{\theta}^2 sin\varphi cos\varphi = a_{T\varphi} - a_{M\varphi} \tag{3}$$

where $a_M = [a_{MR} \quad a_{M\theta} \quad a_{M\varphi}]$ and $a_T = [a_{TR} \quad a_{T\theta} \quad a_{T\varphi}]$ are the accelerations of missile and target. Nullifying the LOS rates, $\dot{\theta}$ and $\dot{\varphi}$, is one of the main objectives of the guidance laws. Eq. (2) and Eq. (3) is used to design the guidance laws.

$$\ddot{\theta} = -\frac{2\dot{R}}{R}\dot{\theta} + 2\dot{\theta}\dot{\varphi}tan\varphi - \frac{a_{M\theta}}{Rcos\varphi} + \frac{a_{T\theta}}{Rcos\varphi} \tag{4}$$

$$\ddot{\varphi} = -\frac{2\dot{R}}{R}\dot{\varphi} - \dot{\theta}^2 sin\varphi cos\varphi - \frac{a_{M\varphi}}{R} + \frac{a_{T\varphi}}{R} \tag{5}$$

where φ must be within (-0.5π 0.5π) as the initial of value. Thus, $cos\varphi > 0$ obtained.

GUIDANCE LAWS DESIGN

The general purpose of the guidance is to implement acceleration commands appropriate to the missile to provide that the distance between the missile and the target reaches zero or minimal value in a finite time. For this purpose, there are many guidance methods designed from past to present, which are called as classical guidance laws, modern guidance laws and robust guidance laws.

In this section, PN and APN guidance laws, Proportional-Integral guidance law and Sliding Mode guidance law are examined in three-dimensional engagement geometry. The block diagram of general missile guidance law design is demonstrated in Figure 2.

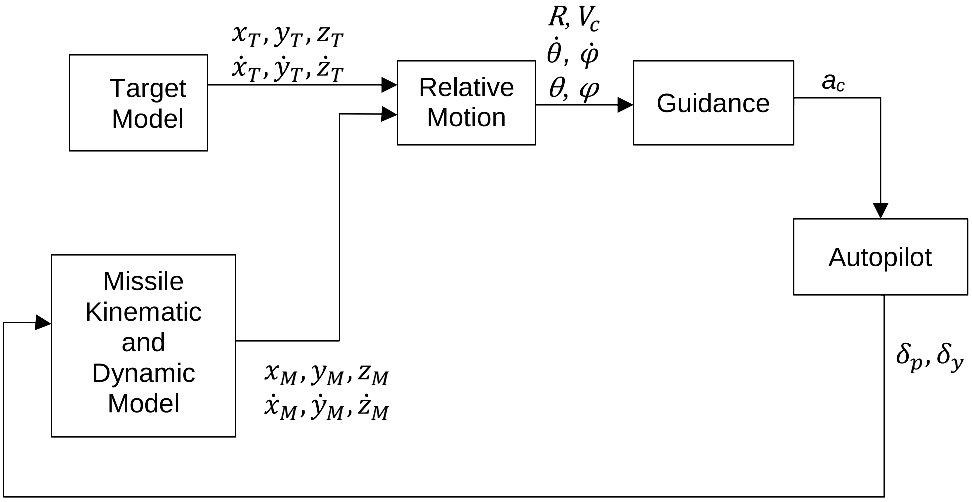

Figure 2. The block diagram of the missile guidance

PN and APN Guidance Laws

PN is the most used in practice guidance approach. The first missile using this law is called the "Lark" missile and the tests of this missile were completed in 1950 successfully. The generated acceleration commands are proportional to the closing speed at which the distance between the missile and the target as well as the rate of change of LOS.

The PN in the general non-linear geometry is demonstrated by

$$a_{my} = NV_c\dot{\theta} \tag{6}$$

$$a_{mz} = NV_c\dot{\varphi} \tag{7}$$

where a_{my} and a_{mz} are the acceleration commands, N is a positive constant, which generally should be selected between 3 and 5 [Zarchan, 2012]. V_c denotes the closing velocity, $\dot{\theta}$ and $\dot{\varphi}$ are the LOS angular rates.

APN is a variation of PN guidance law and its performance has better than the performance of PN when the target is maneuverable. This guidance law needs the target acceleration as well as the closing velocity and the LOS angular rates. The major disadvantage of this guidance law is that it is not always possible to measure or estimate the acceleration of the target.

Mathematically the APN guidance law can be stated as

$$a_{my} = N_1 V_c \dot{\theta} + 0.5 N_2 (a_{\ t\theta}) \tag{8}$$

$$a_{mz} = N_1 V_c \dot{\varphi} + 0.5 N_2 (a_{\ t\varphi}) \tag{9}$$

where $a_{\ t\theta}$ and $a_{\ t\varphi}$ are the target acceleration commands and also N_1, N_2 are a positive constants.

PID Guidance Law

PID controller method is quite important for a wide range of scientific and industrial control processes. Although PID has been used over many years, it remains popular today because of their simple functionality, easy implementation. The control law of the PID method is expressed by:

$$u(t) = K_p e + K_i \int e + K_d \dot{e} \tag{10}$$

where K_p, K_i, K_d are the proportional gain, the integral gain and the derivative gain, respectively. Different controllers can be developed by using one or more of these terms.

The guidance law based on PI control method is designed as follows

$$a_{my} = N_p V_c \dot{\theta} + N_i V_c \theta \tag{11}$$

$$a_{mz} = N_p V_c \dot{\varphi} + N_i V_c \varphi \tag{12}$$

where N_p and N_i are the proportional gain and the integral gain, respectively. V_c denotes the closing velocity, $\dot{\theta}$ and θ is the elevation LOS rate and angle, and also $\dot{\varphi}$ and φ the azimuth LOS rate and angle, respectively.

Sliding Mode Guidance Law

In this subsection, classic SMC is briefly explained and then the proposed sliding mode guidance law is presented in three-dimensional environment. Traditional SMC is a robust control method that has been implemented in almost every control application under uncertainties and disturbances. This control method is composed of two phases. The first phase is to design the switching function in which sliding motion occurs. The other phase is to determine a control law in order to maintain system states on that sliding surface. Thus, u_{sw} is the switching control and u_{eq} is the equivalent control. These are designed as follows.

$$u(t) = u_{eq} + u_{sw} \tag{13}$$

Firstly, the sliding surface is determined to design the sliding mode guidance law. In this paper, two sliding surfaces are designed to zero both LOS angular rate and LOS angle of elevation and of azimuth at the same time. These surfaces as bellows

$$s_1 = \dot{\varphi} \tag{14}$$

$$s_2 = \dot{\theta} cos\varphi \tag{15}$$

For the proposed guidance law, the first time derivative of the sliding surfaces that are expressed by s_1 and s_2 are:

$$\dot{s}_1 = \ddot{\varphi} \tag{16}$$

$$\dot{s}_2 = \ddot{\theta} cos\varphi - \dot{\theta}\dot{\varphi} cos\varphi sin\varphi \tag{17}$$

Consider Lyapunov candidate function is shown by:

$$V = \frac{1}{2}(\dot{\theta}^2 cos^2\varphi + \dot{\varphi}^2) \geq 0 \tag{18}$$

To guarantee a finite time convergence to the sliding surface, $\dot{V} < 0$ must be ensured.

The conclusion of the above condition, the sliding mode guidance law is explained as follows.

$$a_{my} = NV_c\dot{\theta} + \varepsilon_1 sgn(\dot{\theta}) \tag{19}$$

$$a_{mz} = NV_c\dot{\varphi} + \varepsilon_2 sgn(\dot{\varphi}) \tag{20}$$

where $\varepsilon_1 > \|a_{Ty}\|_\infty$ and $\varepsilon_2 > \|a_{Tz}\|_\infty$ are chosen and sgn is the signum function described by

$$sgn(s) = \begin{pmatrix} 1 & if\ s > 0 \\ 0 & if\ s = 0 \\ -1 & if\ s < 0 \end{pmatrix} \tag{21}$$

To minimize the chattering phenomenon, saturation function is used instead of signum function. Consequently, this guidance law is more robust against uncertainties and external disturbances. The sliding mode guidance law is rewritten as below.

$$a_{my} = NV_c\dot{\theta} + \varepsilon_1 sat(\dot{\theta}) \tag{22}$$

$$a_{mz} = NV_c\dot{\varphi} + \varepsilon_2 sat(\dot{\varphi}) \tag{23}$$

SIMULATION RESULTS

In this section, the numerical simulations for the missile-target intercepting in 3D environment is presented. The speed of the target is constant and the target has two different actions, they being non-maneuvering and maneuvering actions. The simulations are performed in MATLAB/Simulink and is used the ode4 (Runge-Kutta) with fixed step size of 0.001 s in the simulation. The system performance is investigated by adding an external disturbance, which is the band limited white noise with the power 25e-5.

The missile and target parameters used in simulation are as follows. The missile initial position is $x_{M0} = 0$ m, $y_{M0} = 5000$ m and $z_{M0} = 0$ m. Its initial velocity is $V_{M0} = 204$ m/s. The target initial position $x_{T0} = 2500$ m, $y_{T0} = 5000$ m and $z_{T0} = 0$ m and its initial velocity is $V_{T0} = 200$ m/s.

The guidance law parameters are taken different for each scenario. For first scenario, the parameter of the PN Eq. (6)-(7) is $N = 4$. The parameters of the APN guidance law are $N_1 = 3$ and $N_2 = 4$. For PI guidance law, the parameters are determined to be $N_p = 5$ and $N_i = 0.73$ in Eq. (11)-(12). Moreover, the guidance law parameters based on SMC method are $N = 4$, $\varepsilon_1 = 10$ and $\varepsilon_2 = 10$. For another scenario, the parameter of PN Eq. (6)-(7) is $N = 4$. The parameters of the APN guidance law are $N_1 = 3$ and $N_2 = 2$. For PI guidance law, the navigation constants are preferred $N_p = 5$ and $N_i = 0.73$ in Eq. (11)-(12). Moreover, the guidance parameters of SM-based law are $N = 4$, $\varepsilon_1 = 10$ and $\varepsilon_2 = 18$ in Eq. (23)-(24).

There are important conditions that must be fulfilled to show that the proposed guidance law is successful. Miss distance should be as small as possible and the LOS rates should be close to zero at finite time. Two simulation scenarios are constructed so as to ensure the performance of the designed guidance laws. Figures 3-16 demonstrate the simulation results. These figures are examined in two parts. The first part is to present between Figure 3 and Figure 9.

It can be clearly seen from in Figure 3 that the relative range decreases to zero at the intercept time in all of the guidance laws. Figure 4 gives the change of Mach number over time. As presented in Figure 5-6, the missile successfully intercepts the target in the implementation of the whole guidance laws. However, the presented PI-based guidance law follows a different trajectory. From Figure 7, it can be observed that all of them can verify that LOS angular rates $\dot{\theta}$ and $\dot{\varphi}$ decrease to zero in finite time. And also in Figure 8 it is shown that LOS angles become stable at zero in finite time. Finally, two figures related to sliding mode based guidance law are examined in Figure 9-10. These figures are observed that the sliding surfaces become stable at zero in finite time. Thus, the sliding surface variables are understood to be smooth and stable.

Miss distances and final times are given for non-maneuvering target in Table 1. It is clearly observed that the presented guidance law yields successful results in terms of miss distances and target tracking ability.

Table 1. Miss distances and final times for non-maneuvering target

Sc-1 (Non- maneuvering target)	Miss distance	Time
PN Guidance law	0.8511	9.7480
APN Guidance law	0.9372	9.7480
PI Guidance law	0.7955	9.7480
SM Guidance law (sat)	0.8491	9.7480

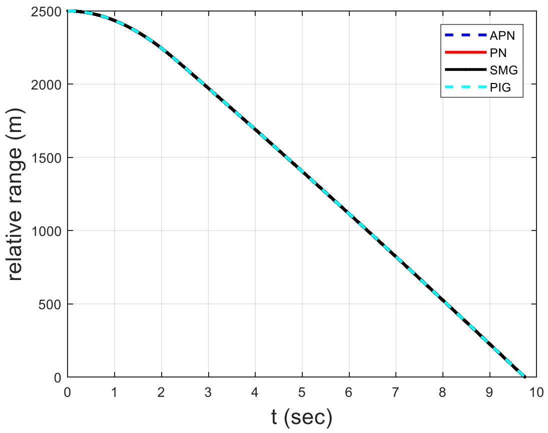

Figure 3. Relative Range

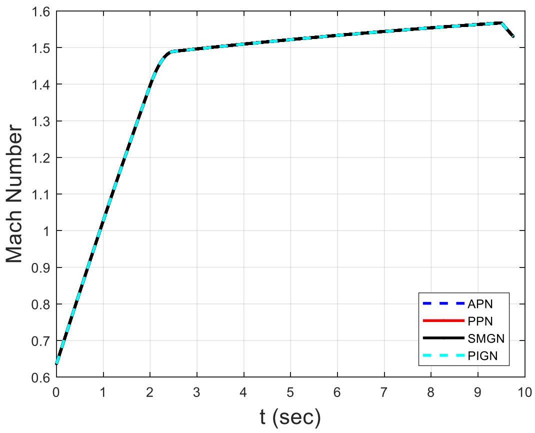

Figure 4. Mach Number

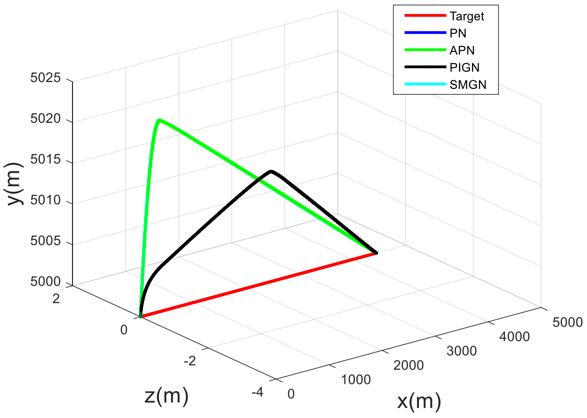

Figure 5. Missile and Target trajectories for all guidance laws

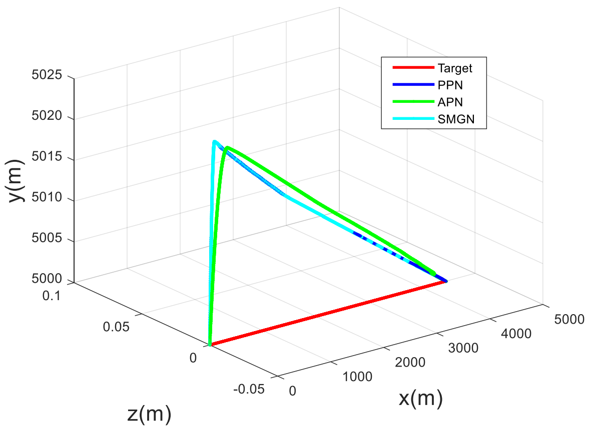

Figure 6. Missile and Target trajectories for PN, APN, SMG

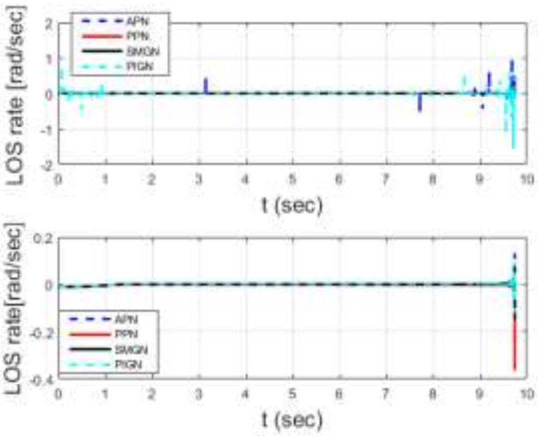

Figure 7. Elevation and azimuth angular rates of LOS

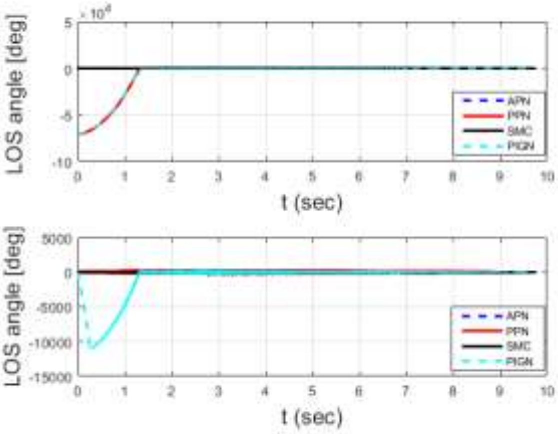

Figure 8. Elevation and azimuth angels of LOS

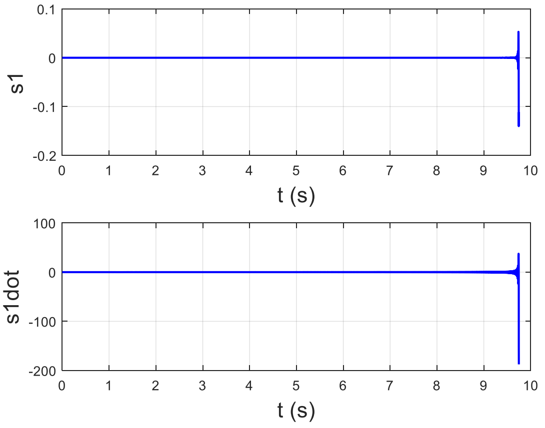

Figure 9. Sliding mode manifold of s_1 and $s_1$dot

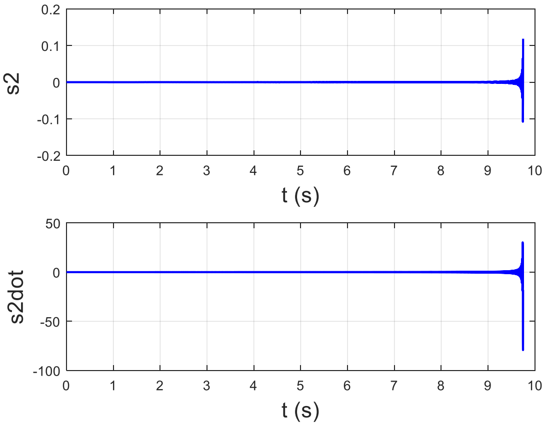

Figure 10. Sliding mode manifold of s_2 and $s_2$dot

In the second part, the results of the guidance rules against a maneuvering target are demonstrated between Figure 11 and Figure 18. The relative range presented in Figure 11 declines to zero at the intercept time, as expected from all of the guidance laws. Figure 12 presents the change of Mach number over time. In Figure 13-14, the missile successfully intercepted the target for each guidance laws presented in this paper. As shown in Figure 15 and Figure 16, $\dot{\theta}$ and $\dot{\varphi}$ which are known as LOS angular rates converge to zero and the same can be said for the LOS angles. In addition, the sliding surfaces have become stable at zero as in the first scenario for the SM-based guidance law. It is observed in Figure 17-18 that the presented guidance law is stable and smooth when the target is capable of maneuvering.

For the maneuvering target, miss distances and final times are presented in Table 2. It can be clearly observed that the designed guidance law yields successful results in terms of miss distances as well as target tracking ability.

Table 2. Miss distances and final times for maneuvering target

Sc-2 (Maneuvering target)	Miss distance	Time
PN Guidance Law	0.8598	9.6040
APN Guidance Law	0.8111	9.6040
PI Guidance law	0.8369	9.6040
SM Guidance law (sat)	0.7601	9.6040

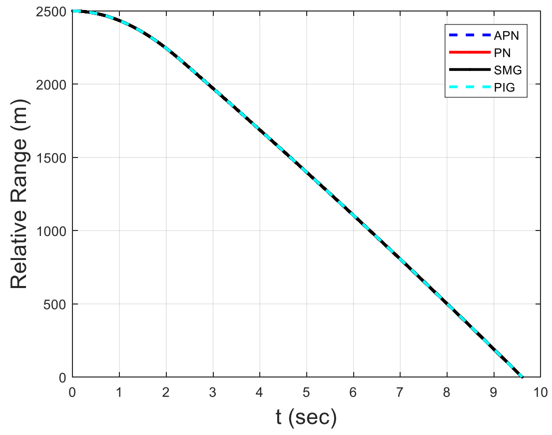

Figure 11. Relative Range

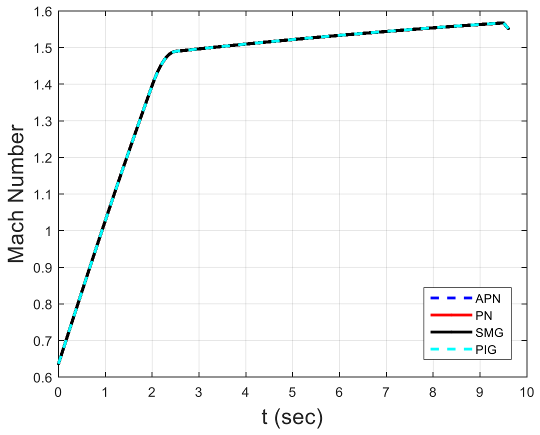

Figure 12. Mach Number

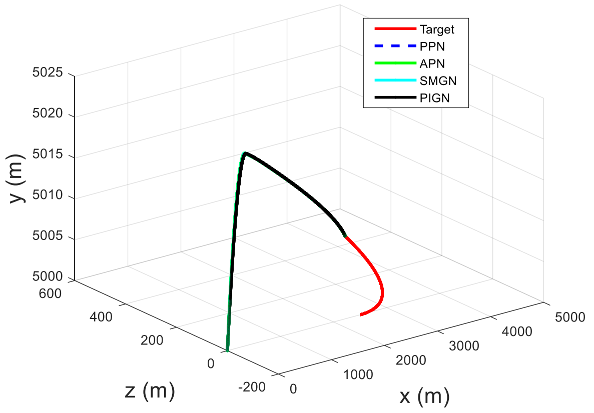

Figure 13. Missile and Target trajectories for all guidance laws

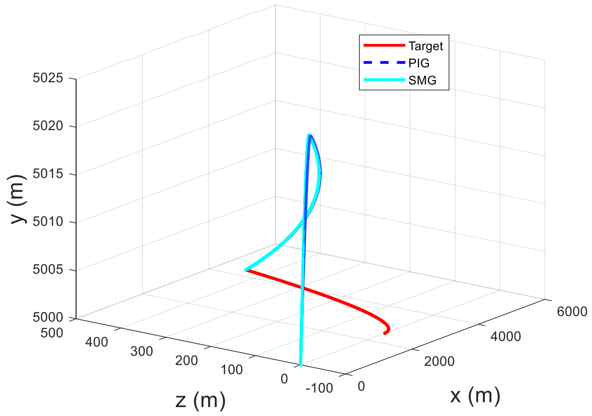

Figure 14. Missile and Target trajectories for SMG and PIG from other angle

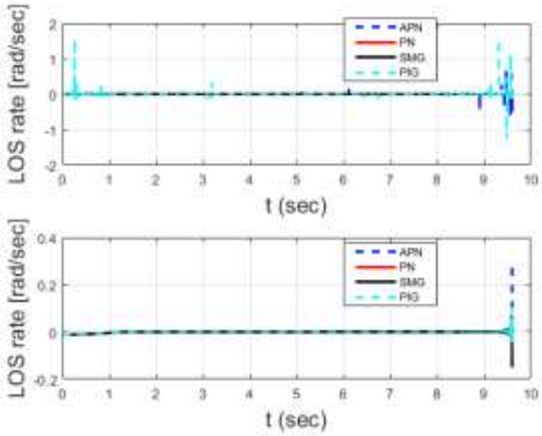

Figure 15. Elevation and azimuth angular rate of LOS

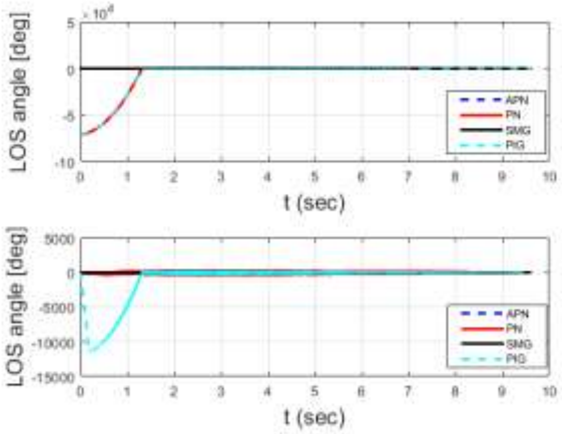

Figure 16. Elevation and azimuth angles of LOS

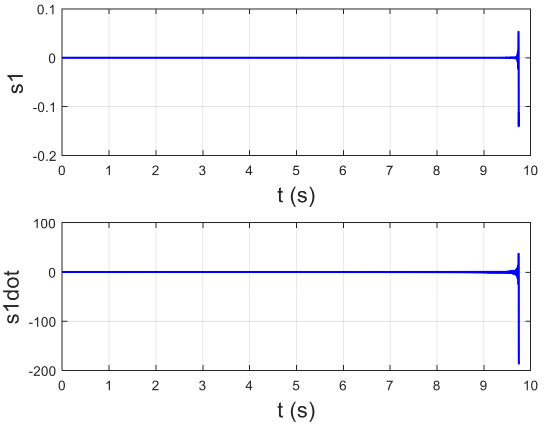

Figure 17. Sliding mode manifold of s_1 and $s_1$dot

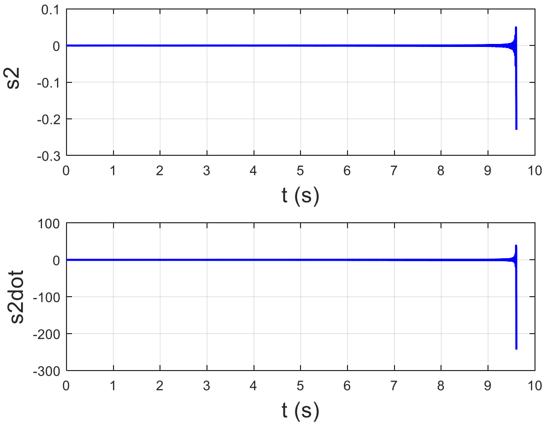

Figure 18. Sliding mode manifold of s_2 and $s_2$dot

CONCLUSIONS

In this paper, the proportional navigation, the augmented proportional navigation, the proportional-integral-derivative guidance law as well as the sliding mode guidance law are investigated by taking different two scenarios into account. The purpose is to design a missile guidance law that intercepts the target in the three-dimensional environment with less sensitivity to the external disturbances. The simulation studies are presented according to two different models of target: non-maneuverable and maneuverable target. It is clearly understood from the results of numerical simulations that the designed control methods based guidance laws are able to give better results than the classical guidance laws in terms of miss distance in all two target model. Furthermore, the relative range, the LOS angles as well as the LOS angular rates converge to zero in finite time as expected from all of the guidance laws presented.

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