VSS Theory Based Training of a Fuzzy Motion Control System

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Abstract

This paper presents a novel training algorithm for adaptive neuro-fuzzy inference systems. The algorithm combines the error backpropagation algorithm with variable structure systems approach. Expressing the parameter update rule as a dynamic system in continuous time and applying sliding mode control (SMC) method to the dynamic model of the gradient based training procedure results in the parameter stabilizing part of training algorithm. The combination therefore leads to the minimization of parametric displacements together with a considerable improvement on tracking performance. In the application example, control of a two degrees of freedom direct drive SCARA robotic manipulator is considered. As the controller, an adaptive neuro-fuzzy inference mechanism is used, and in the parameter tuning, the proposed algorithm is utilized.

1. Introduction

Soft computing is a practical alternative for solving complex problems through the use of human expertise and a priori knowledge about the problem in hand. *Fuzzy Inference Systems* are the most popular constituent of the soft computing area because of their ability to represent human expertise in the form of *IF antecedent THEN consequent* statements. In this domain, the system behavior is modeled through the use of linguistic descriptions.

The typical architectures of fuzzy inference systems are those introduced by Wang [1,2], Takagi and Sugeno [3], and Jang [4]. In [2], a fuzzy system having Gaussian membership functions, product inference rule and weighted average defuzzifier is constructed and has become the standard method in most applications. Takagi and Sugeno change the defuzzification procedure where dynamic systems are introduced as defuzzification subsystems. The potential advantage of the method is that, under certain constraints, the stability of the system can be studied. Jang *et al* [4] propose an adaptive neuro fuzzy inference system, in which a polynomial is used as the defuzzifier. This structure is commonly referred to as ANFIS in the related literature. The choice concerning the order of the polynomial and the variables to be used in the defuzzifier are left to the designer.

In control engineering practice, stability and robustness are of crucial importance. Because of this, a control engineering expert is always in pursuit of a design, which provide accuracy as well as insensitivity to environmental disturbances and structural uncertainties. A suitable way of tackling with uncertainties and disturbances is to introduce Variable Structure Systems (VSS) theory based components into the system structure.

Numerous contributions to VSS theory have been made during the last decade, some of them are as follows: Hung *et al* [6] have reviewed the control strategy for linear and nonlinear systems. In [6], the switching schemes, putting the differential equations into canonical forms and generating simple SMC strategies are considered in detail. Gao *et al* [7], apply the SMC scheme to robotic manipulators and discuss the quality of the scheme. One of the crucial points in SMC is the selection of the parameters of the sliding surface. Some studies devoted to the adaptive design of sliding surfaces have shown that the performance of control system can be refined by interfacing it with an adaptation mechanism, which regularly redesigns the sliding surface [8]. This eventually results in a robust control system. The performance of SMC scheme is proven to be satisfactory in the face of external disturbances and uncertainties in the system model representation. The latest studies consider this robustness property by equipping the system with computationally intelligent methods. In [9] and [10], fuzzy inference systems are proposed for SMC scheme. A standard fuzzy system is studied and the relevant robustness analyses are carried out. Particularly, the work presented in [9] emphasizes that the robustness and stability properties of soft computing based control strategies can be studied through the use SMC theory. It is shown in the paper in this way that the approach is robust i. e. it can compensate the deficiencies caused by poor modeling of plant dynamics and external disturbances.

This paper is organized as follows: The second section summarizes the conventional method followed in gradient based optimization technique. The third section presents the derivation of SMC based parameter stabilizing law. In the fourth section, ANFIS architecture is considered and the relevant formulation for the approach is given. Next section is devoted to the plant to be controlled in this study. This is followed by the simulation studies, and conclusions constitute the last part of the paper.

2. An Overview of Gradient Descent

In this section, a widely used technique of parameter adjustment is briefly reviewed. The method has first been formulated by Rumelhart *et al* [11] in 1980s. The approach has successfully been applied to a wide variety of optimization problems. The algorithm can briefly be stated as follows. The error in (1) is used to minimize the cost function in (2) by utilizing the rule described by (3). The minimization proceeds iteratively as given in (4), for which the sensitivity derivative with respect to the generic parameter ϕ is needed. It is apparent that the method is applicable to the architectures in which the outputs are differentiable with respect to the subject of optimization.

$$e = d - F(\phi) \tag{1}$$

$$J = \frac{1}{2}e^2 \tag{2}$$

$$\Delta\phi(k) = -\eta\frac{\partial J}{\partial \phi} \tag{3}$$

$$\Delta\phi(k) = \eta e\frac{\partial F(\phi)}{\partial \phi} \tag{4}$$

3. Derivation of the Stabilizing Criteria

A continuous-time dynamic model of the parameter update procedure of (4) can be constructed as described by (5).

$$\dot{\Delta\phi} = -\frac{1}{T_s}\Delta\phi + \frac{\eta_\phi}{T_s}N_\phi \tag{5}$$

If the model in (5) can easily be validated at the integer multiples of the sampling period T_s by replacing the derivative term with Euler's first order approximation. In (5), the evaluated parameter change, which is by error backpropagation and denoted by N_ϕ, is multiplied by a scaling factor denoted by η_ϕ, for the selection of which, a detailed analysis is presented in the subsequent discussion. In the design of variable structure controllers, one method that can be followed is the reaching law approach [6]. For the use of this theory in the stabilization of the training dynamics, let us define the switching function as in (6) and its dynamics as in (7).

$$s_\phi = \Delta\phi \tag{6}$$

$$\dot{s}_\phi = -\frac{Q_\phi}{T_s}\tanh\left(\frac{s_\phi}{\varepsilon}\right) - \frac{K_\phi}{T_s}s_\phi = \dot{\Delta\phi} \tag{7}$$

In above, Q_ϕ and K_ϕ are the gains and ε is the width of the boundary layer. Equating (7) and (5) and solving for $\Delta\phi$ yields the following solution.

$$\Delta\phi = \eta_\phi N_\phi + Q_\phi \tanh\left(\frac{\Delta\phi}{\varepsilon}\right) + K_\phi\Delta\phi \tag{8}$$

In the derivations presented below, a key point is the fact that the system described by (5) is also driven by η_ϕ, which is known as learning rate in the related literature. Now we demonstrate that some special selection of this quantity leads to the minimization of the magnitude of parametric displacements. Let us define the following quantity for keeping analytic comprehensibility;

$$A_\phi = Q_\phi \tanh\left(\frac{\Delta\phi}{\varepsilon}\right) + K_\phi\Delta\phi \tag{9}$$

Now we have a model described by (5), and a solution formulated by (8). If one chooses a positive definite Lyapunov function as given by (10), the time derivative of this function must be negative definite for stability of parameter change ($\Delta\phi$) dynamics. Clearly the stability in parameter change space implies the convergence in system parameters.

$$V_\phi = \frac{1}{2}s_\phi^2 = \frac{1}{2}(\Delta\phi)^2 \tag{10}$$

$$\dot{V}_\phi = (\Delta\phi)(\dot{\Delta\phi}) \tag{11}$$

If (5) and (8) are substituted into (11), the following selection of η_ϕ introduces the stability in the Lyapunov sense.

$$\eta_\phi = \beta \min\left\{\left|\frac{1}{N_\phi}\Delta\phi\right|, \left|-\frac{1}{N_\phi}A_\phi\right|\right\}, \quad 0<\beta<1 \qquad (12)$$

By substituting the learning rate formulated in (12) into the stabilizing solution given in (8), the stabilizing component $\Delta\phi_{VSS}$ of the parameter change formula is obtained as;

$$\Delta\phi_{VSS} = \beta \min\left(|\Delta\phi|, |A_\phi|\right)\mathrm{sgn}\left(N_\phi\right) + A_\phi \qquad (13)$$

where, $\Delta\phi$ on the right hand side is the final update value yet to be obtained. The law introduced in (13) minimizes the cost of stability for parameter ϕ, which is the Lyapunov function defined by (10). The question now reduces to the following; can the cost defined by (2) be minimized by this rule? The answer is obviously not, because the stabilizing criteria is derived from the displacement of a generic parameter denoted by $\Delta\phi$, whereas the minimization of (2) is achieved when ϕ tends to ϕ^* regardless of what the displacement is. Therefore the rule formulated in (13) needs a final modification. In order to minimize (2), the parameter change anticipated by gradient based optimization technique, which is reviewed in the second section, should somehow be integrated into the final form of parameter update mechanism. As introduced in the second section, error backpropagation algorithm (EBP) evaluates a parameter change as given by (14).

$$\Delta\phi_{EBP} = \zeta N_\phi \qquad (14)$$

where, ζ is the constant learning rate in the conventional sense. Combining the laws formulated in (13) and (14) in a weighted average, the eventual form of parameter update law is obtained as given in (15).

$$\Delta\phi = \frac{\alpha_1\Delta\phi_{VSS} + \alpha_2\Delta\phi_{EBP}}{\alpha_1 + \alpha_2} \qquad (15)$$

The parameter update formula given in (15) carries mixed information containing both the parametric convergence, which is introduced by VSS part, and the cost minimization, which is due to the error backpropagation technique. The balancing in this mixture is left to the designer by an appropriate selection of α_1 and α_2.

4. Application to ANFIS

Adaptive Neuro-Fuzzy Inference Systems are realized by an appropriate combination of neural and fuzzy systems. This paper considers the ANFIS structure with first order Sugeno model containing nine rules. The structure for two inputs and one output is illustrated in Fig. 1 and a sample rule is described below for a m-input one output ANFIS.

IF *u_1 is $U_{i,1}$ AND u_2 is $U_{i,2}$ AND … AND u_m is $U_{i,m}$*
THEN *$f_i = q_{i,1}u_1 + \ldots + q_{i,m}u_m + q_{i,m+1}$*

In the IF part of this representation, lowercase variables denote the inputs, uppercase variables stand for the fuzzy sets corresponding to the domain of each linguistic label. The ANFIS output is clearly a linear function of the adjustable defuzzifier parameters denoted by $q_{i,j}$. The system that is considered in this study uses Gaussian membership functions as described by (16) and the overall realization is given in (17).

$$\mu_{ij}\left(u_j\right) = \exp\left\{-\left(\frac{u_j - c_{ij},}{\sigma_{ij}}\right)^2\right\} \qquad (16)$$

$$F = \frac{\sum_{i=1}^{\#Rules} f_i \prod_{j=1}^{\#Inputs} \mu_{ij}\left(u_j\right)}{\sum_{i=1}^{\#Rules} \prod_{j=1}^{\#Inputs} \mu_{ij}\left(u_j\right)} = \sum_{i=1}^{\#Rules} f_i w_{ni} \qquad (17)$$

In (17), the vector of firing strengths denoted by w is normalized and the resulting vector is represented by w_n. The relevant backpropagated error values for the adjustable ANFIS parameters are given in (18) through (20).

$$N_{q_{i,j}} = \begin{cases} e w_{ni} u_j & 1 \le j \le m+1 \\ e w_{ni} & j = m+1 \end{cases} \qquad (18)$$

$$N_{c_{ij}} = e\left(f_i - F\right) w_{ni} 2 \frac{u_j - c_{ij}}{\sigma_{ij}^2} \qquad (19)$$

$$N_{\sigma_{ij}} = e\left(f_i - F\right) w_{ni} 2 \frac{\left(u_j - c_{ij}\right)^2}{\sigma_{ij}^3} \qquad (20)$$

The error backpropagation based part of the training procedure is evaluated by using the quantities described in (18) through (20). The form of this part is given by (14). The final form of the mixed training information can now be constructed as formulated in (15).

5. Plant Model

In this study, a two degrees of freedom direct drive robotic manipulator is used as the test bed. Since the dynamics of such a mechatronic system is modeled by nonlinear and coupled differential equations, precise output tracking becomes a difficult objective due to the strong interdependency between the variables involved. Furthermore, the ambiguities concerning the friction related dynamics in the plant model make the design much more complicated. Therefore the methodology adopted must use the methods of computational intelligence in some sense.

The general form of robot dynamics is described by (21) where M, V, τ and f_c stand for the state varying inertia matrix, vector of coriolis terms, applied torque inputs and friction terms respectively.

$$M(\theta)\ddot{\theta}+V(\theta,\dot{\theta})=\tau-f_c \tag{21}$$

If the angular positions and angular velocities are defined as the state variables of the system, four coupled and first order differential equations can define the model in state space. In (22) and (23), the terms seen in (21) are given explicitly.

$$M(\theta)=\begin{bmatrix} p_1+2p_3cos(\theta_2) & p_2+p_3cos(\theta_2) \\ p_2+p_3cos(\theta_2) & p_2 \end{bmatrix} \tag{22}$$

$$V(\theta,\dot{\theta})=\begin{bmatrix} -\dot{\theta}_2\left(2\dot{\theta}_1+\dot{\theta}_2\right)p_3\sin(\theta_2) \\ \dot{\theta}_1^2 p_3\sin(\theta_2) \end{bmatrix} \tag{23}$$

In above, p_1 = 2.0857, p_2 = 0.1168 and p_3 = 0.1630. The details of the plant model are presented in [12].

6. Simulation Studies

In the simulation studies presented, the manipulator is controlled by two ANFIS controllers. The main objective is to keep the update dynamics in a stable region. This is achieved through a suitable combination of gradient based optimization technique and the strategy based on the variable structure systems approach. The control system is illustrated in Fig. 2. The reference velocity in (24) is used in all simulations with zero initial errors.

$$\dot{\theta}_{d\,1,2}=\sin\left(\frac{2\pi\, t}{5}\right) \tag{24}$$

The results presented concern the tuning of all adjustable parameters of the ANFIS structure during the learning process. The state tracking errors are depicted in Fig. 3. It is evident from Fig. 3 that once a fluctuation occurs on the error or rate of error, it is dampened out by the use of VSC philosophy in the learning strategy. In the simulations discussed, the settings used are tabulated in Table 1.

In the training of the controllers, the squared sum of parametric changes can be defined to be the total cost of stability, which is described by (25).

$$J_s(t)=\sum_{\phi}\left(\Delta\phi(t)\right)^2 \tag{25}$$

In Fig. 4, the cost of tracking described by (2) is illustrated whereas the bottom row depicts the time behavior of the parametric cost described by (25).

Table 1. The Settings Used in the Simulations

T_s	2.5 msec.	Q	0.1
β	0.1	K	0.1
ζ	0.02	ε	1.0
α_{1i}	3.0 $\forall i$	*#Rules*	9
α_{2i}	2.0 $\forall i$	#AN*FIS Inputs*	2

As can be inferred from Fig. 4, both the parametric stabilization and the tracking error minimization performance of the proposed methodology is highly promising. Another remarkable property of the algorithm presented is that it operates on-line. Therefore, the difficulties that are likely to occur in on-line learning and control are alleviated by the robustness provided by the integration with VSS technique.

7. Conclusions

In this paper, a novel technique for improving learning performance of adaptive neuro-fuzzy inference systems is presented. An approximate dynamic model of the error backpropagation procedure is constructed and variable structure systems approach is incorporated into the model of the parameter update law. In this procedure, gradient descent method is responsible for the minimization of squared error while the variable structure systems based law is responsible for the stability in the parameter space.

In the application example presented, the results confirm the prominent features of the approach, namely the parametric stability and tracking performance improvement. The algorithm is applicable to any neuro-fuzzy system model provided that the model output is differentiable with respect to the parameter of interest.

8. References

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9. Acknowledgments

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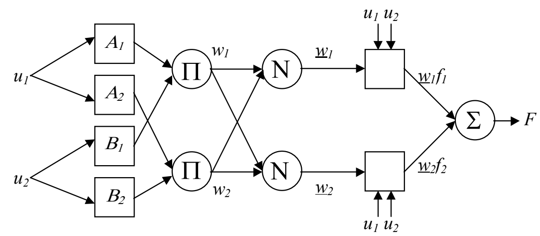

Figure 1. Architecture of ANFIS

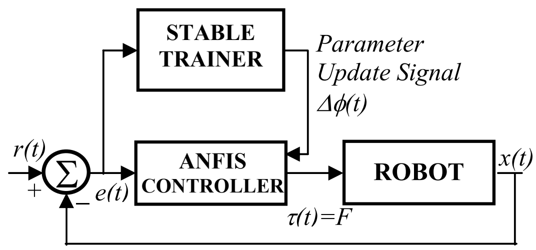

Figure 2. Control System Architecture

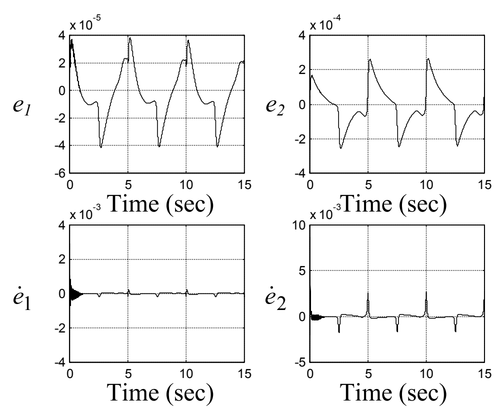

Figure 3. State Tracking Error Graph

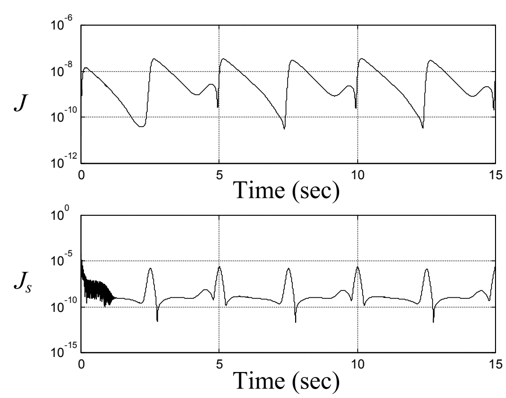

Figure 4. Time Behavior of the Parametric Cost for Base and Elbow Controllers Respectively