Harvard University Extension School
Computer Science E-207

Problem Set 2 CORRECTED

Due Friday, September 28, 2012 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps2 CORRECTED.pdf emailed to cscie207@seas.harvard.edu.

LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

Unless specified otherwise, assume the alphabet $\Sigma = \{a, b\}$

PROBLEM 1 (2+2+2+2+2 points)

Translate the following languages from set notation with English description to regular expressions, or vice versa:

(A) $((a \cup b)^*a) \cup ((a \cup b)^*b)$
(B) $(((a \cup b)a)^* \cup ((a \cup b)b)^*)(a \cup b)$
(C) $L = \{w \in \Sigma^* : w \text{ has no more than 2 } b\text{'s}\}$
(D) $L = \{w : w \text{ has length } 3\}$
(E) $L = \{w \in \Sigma^* : w \text{ has no consecutive b's}\}$

PROBLEM 2 (6+6 points)

(A) Prove that if R is a regular expression that contains no occurrences of *, then $L(R)$ is finite. Note: You may assume (without proof) basic facts about finite sets for this problem; however, *explicitly* state any such assumptions you make. [Hint: use structural induction on R.]
(B) Prove or disprove: every regular expression that does contain an occurrence of * generates an infinite language.

PROBLEM 3 (12 points)

(A) Construct a DFA for $L = \{w \in \Sigma^* : \text{there are an even number of } a\text{'s and an even number of } b\text{'s in } w\}$

(B) Convert your DFA for L to a regular expression using the GNFA construction described in lecture and in Sipser (p. 66, Lemma 1.60 in the Second Edition). Show the steps of the construction. As you go along, use basic simplifications, such as $(a \cup \varepsilon)^* \to a^*$ to make the REs simpler.

PROBLEM 4 (1+1+10 points)

Let Σ and Δ be alphabets. Consider a function $\psi : \Sigma \to \Delta^*$. Extend ψ to a function from $\Sigma^* \to \Delta^*$ using the inductive rules:

$$\psi(\varepsilon) = \varepsilon$$
$$\psi(w\sigma) = \psi(w)\psi(\sigma) \text{ , for any } w \in \Sigma^*, \sigma \in \Sigma$$

(A) For example, consider $\Sigma = \{a, b\}$ and $\Delta = \{a, b, c, d\}$ and let $\psi(a) = ccb$, $\psi(b) = dda$. What is $\psi(aba)$?

(B) Any function $\psi : \Sigma^* \to \Delta^*$ defined from a function $\psi : \Sigma \to \Delta^*$ in this way is termed a *homomorphism*.

Now for a language L and homomorphism ψ, define

$$\psi(L) = \{\psi(w) : w \in L\}$$

For example, we can extend ψ as defined in (A) to languages. If $L = \{bb, aa, ba\}$, then what is $\psi(L)$ in this case?

(C) Prove that the set of regular languages is closed under homomorphism. That is, if L is regular and ψ is a homomorphism, then $\psi(L)$ is regular.

PROBLEM 5 (Challenge! 2 points)

Let $L\backslash A = \{x : wx \in A \text{ for some } w \in L\}$. Show that if A is regular and L is *any* language, then $L\backslash A$ is regular.