# Introduction to Information Retrieval

http://informationretrieval.org

## IIR 21: Link Analysis

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# Overview


1. Recap
2. Anchor text
3. Citation analysis
4. PageRank
5. HITS: Hubs & Authorities

# Outline

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# Applications of clustering in IR

| Application | What is clustered? | Benefit | Example |
|---|---|---|---|
| Search result clustering | search results | more effective information presentation to user | |
| Scatter-Gather | (subsets of) collection | alternative user interface: "search without typing" | |
| Collection clustering | collection | effective information presentation for exploratory browsing | McKeown et al. 2002, news.google.com |
| Cluster-based retrieval | collection | higher efficiency: faster search | Salton 1971 |

# $K$-means algorithm

$K$-MEANS$(\{\vec{x}_1, \ldots, \vec{x}_N\}, K)$

$(\vec{s}_1, \vec{s}_2, \ldots, \vec{s}_K) \leftarrow$ SELECTRANDOMSEEDS$(\{\vec{x}_1, \ldots, \vec{x}_N\}, K)$
**for** $k \leftarrow 1$ **to** $K$
**do** $\vec{\mu}_k \leftarrow \vec{s}_k$
**while** stopping criterion has not been met
**do for** $k \leftarrow 1$ **to** $K$
    **do** $\omega_k \leftarrow \{\}$
    **for** $n \leftarrow 1$ **to** $N$
    **do** $j \leftarrow \arg\min_{j'} |\vec{\mu}_{j'} - \vec{x}_n|$
        $\omega_j \leftarrow \omega_j \cup \{\vec{x}_n\}$ *(reassignment of vectors)*
    **for** $k \leftarrow 1$ **to** $K$
    **do** $\vec{\mu}_k \leftarrow \frac{1}{|\omega_k|} \sum_{\vec{x} \in \omega_k} \vec{x}$ *(recomputation of centroids)*
**return** $\{\vec{\mu}_1, \ldots, \vec{\mu}_K\}$

# Initialization of $K$-means

- Random seed selection is just one of many ways $K$-means can be initialized.
- Random seed selection is not very robust: It's easy to get a suboptimal clustering.
- Better heuristics:
  - Select seeds not randomly, but using some heuristic (e.g., filter out outliers or find a set of seeds that has "good coverage" of the document space)
  - Use hierarchical clustering to find good seeds (next class)
  - Select $i$ (e.g., $i = 10$) different sets of seeds, do a $K$-means clustering for each, select the clustering with lowest RSS

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- Citation analysis: the mathematical foundation of PageRank and link-based ranking
- PageRank: the original algorithm that was used for link-based ranking on the web
- Hubs & Authorities: an alternative link-based ranking algorithm

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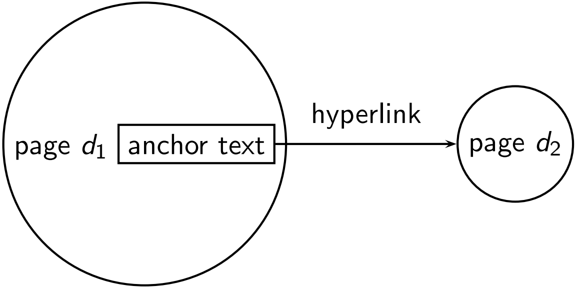

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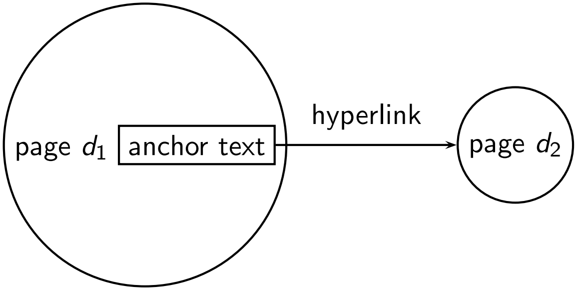


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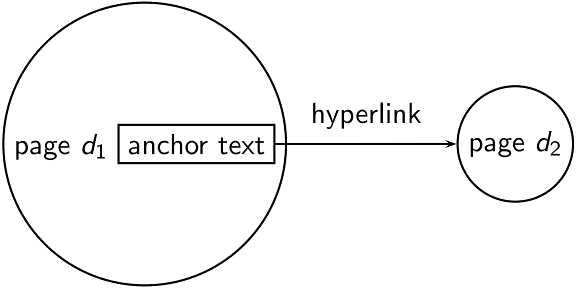


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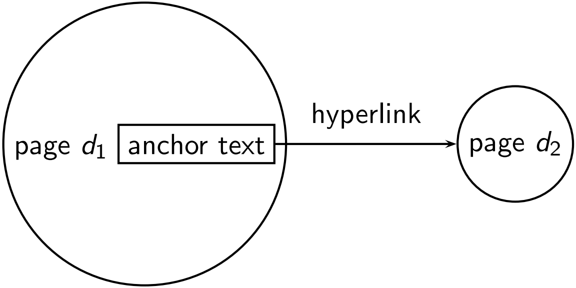


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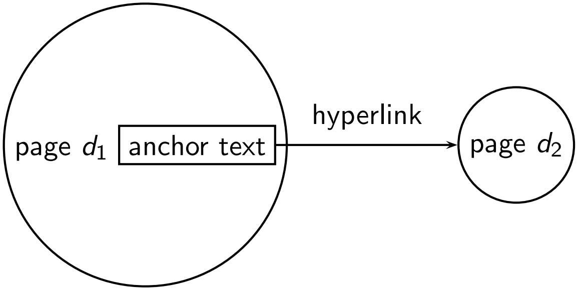


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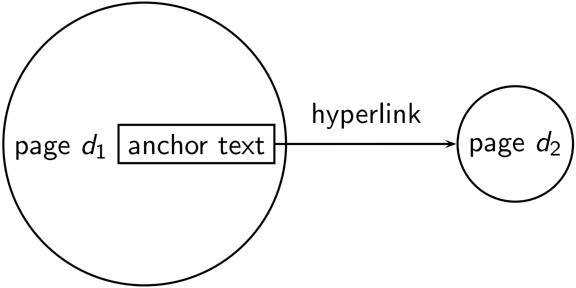


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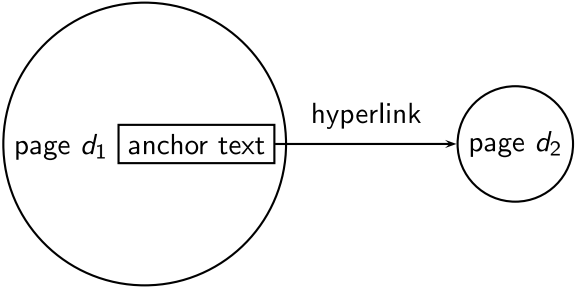


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  - ... if IBM home page is mostly graphics
- Searching on [anchor text $\rightarrow d_2$] is better for the query *IBM*.
  - In this representation, the page with the most occurrences of *IBM* is www.ibm.com. □

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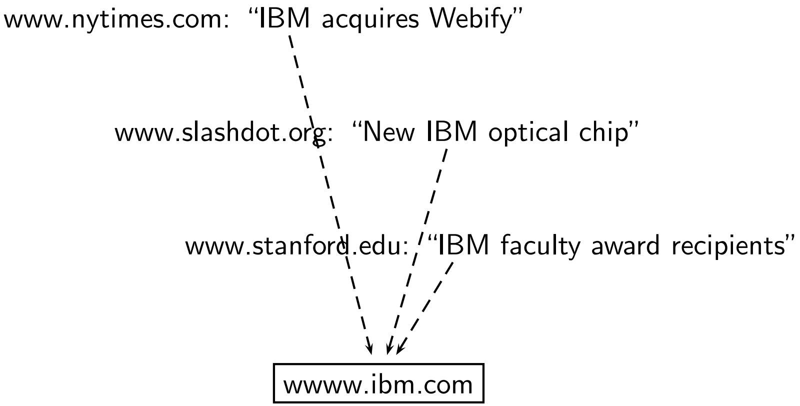

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- Anchor text can be weighted more highly than document text. (based on Assumptions 1&2)

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- Is assumption 2 true in general?

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- Defused Google bombs: [dumb motherf....], [who is a failure?], [evil empire]

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  - Cocitation similarity on the web: Google's "related:" operator, e.g. [related:www.ford.com]

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  - Circular? No: can be formalized in a well-defined way.

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- Citation analysis is a big deal: The budget and salary of this lecturer are / will be determined by the impact of his publications!

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- Next: PageRank algorithm for computing weighted citation frequency on the web □

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- This long-term visit rate is the page's PageRank.
- PageRank = long-term visit rate = steady state probability □

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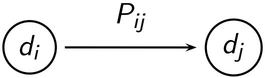
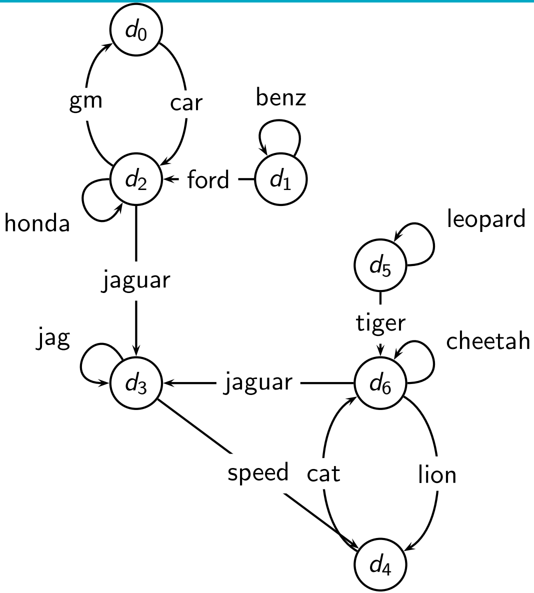
- At each step, we are on exactly one of the pages.
- For $1 \leq i, j \leq N$, the matrix entry $P_{ij}$ tells us the probability of $j$ being the next page, given we are currently on page $i$.
- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$



# Example web graph



# Link matrix for example

| | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $d_1$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| $d_2$ | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| $d_3$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $d_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $d_5$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $d_6$ | 0 | 0 | 0 | 1 | 1 | 0 | 1 |

# Transition probability matrix $P$ for example

| | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $d_1$ | 0.00 | 0.50 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 |
| $d_2$ | 0.33 | 0.00 | 0.33 | 0.33 | 0.00 | 0.00 | 0.00 |
| $d_3$ | 0.00 | 0.00 | 0.00 | 0.50 | 0.50 | 0.00 | 0.00 |
| $d_4$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| $d_5$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.50 |
| $d_6$ | 0.00 | 0.00 | 0.00 | 0.33 | 0.33 | 0.00 | 0.33 |

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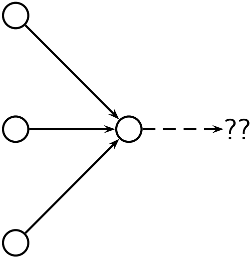
- Next: what properties must hold of the web graph for the long-term visit rate to be well defined?
- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends. □

# Dead ends



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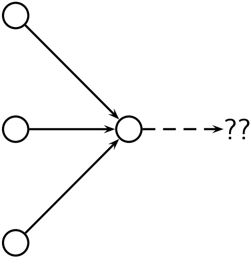


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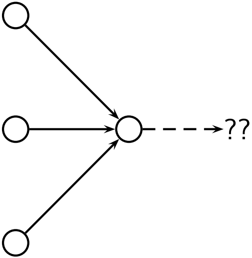


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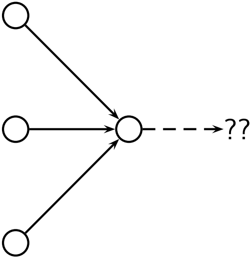


- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

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- More generally, we require that the Markov chain be ergodic.

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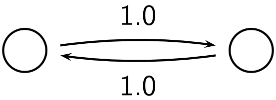
- A non-ergodic Markov chain:



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- Teleporting makes the web graph ergodic.
- ⇒ Web-graph+teleporting has a steady-state probability distribution.
- ⇒ Each page in the web-graph+teleporting has a PageRank.

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- Next: how to compute PageRank

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- $\sum x_i = 1$

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- So from $\vec{x}$, our next state is distributed as $\vec{x}P$. □

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- So we can think of PageRank as a very long vector – one entry per page.

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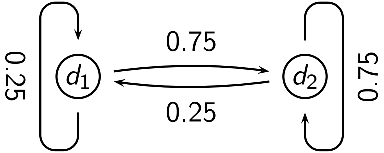

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- All transition probability matrices have largest eigenvalue 1. □

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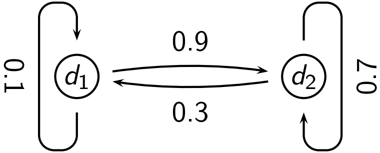

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| $t_1$ | | | | | $= \vec{x}P^2$ |
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| | | | | | ... |
| $t_\infty$ | | | | | $= \vec{x}P^\infty$ |

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| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | | | | | $= \vec{x}P^3$ |
| $t_3$ | | | | | $= \vec{x}P^4$ |
| | | | | | ... |
| $t_\infty$ | | | | | $= \vec{x}P^\infty$ |

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

# Computing PageRank: Power method

| | $x_1$ $P_t(d_1)$ | $x_2$ $P_t(d_2)$ | | | |
|---|---|---|---|---|---|
| | | | $P_{11} = 0.1$ $P_{21} = 0.3$ | $P_{12} = 0.9$ $P_{22} = 0.7$ | |
| $t_0$ | 0 | 1 | 0.3 | 0.7 | $= \vec{x}P$ |
| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | 0.24 | 0.76 | | | $= \vec{x}P^3$ |
| $t_3$ | | | | | $= \vec{x}P^4$ |
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| $t_0$ | 0 | 1 | 0.3 | 0.7 | $= \vec{x}P$ |
| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | 0.24 | 0.76 | 0.252 | 0.748 | $= \vec{x}P^3$ |
| $t_3$ | | | | | $= \vec{x}P^4$ |
| | | | | | ... |
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| $t_2$ | 0.24 | 0.76 | 0.252 | 0.748 | $= \vec{x}P^3$ |
| $t_3$ | 0.252 | 0.748 | | | $= \vec{x}P^4$ |
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| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | 0.24 | 0.76 | 0.252 | 0.748 | $= \vec{x}P^3$ |
| $t_3$ | 0.252 | 0.748 | 0.2496 | 0.7504 | $= \vec{x}P^4$ |
| | | | | | ... |
| $t_\infty$ | | | | | $= \vec{x}P^\infty$ |

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| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | 0.24 | 0.76 | 0.252 | 0.748 | $= \vec{x}P^3$ |
| $t_3$ | 0.252 | 0.748 | 0.2496 | 0.7504 | $= \vec{x}P^4$ |
| | | | ... | | ... |
| $t_\infty$ | 0.25 | 0.75 | | | $= \vec{x}P^\infty$ |

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| $t_0$ | 0 | 1 | 0.3 | 0.7 | $= \vec{x}P$ |
| $t_1$ | 0.3 | 0.7 | 0.24 | 0.76 | $= \vec{x}P^2$ |
| $t_2$ | 0.24 | 0.76 | 0.252 | 0.748 | $= \vec{x}P^3$ |
| $t_3$ | 0.252 | 0.748 | 0.2496 | 0.7504 | $= \vec{x}P^4$ |
| | | | | ... | ... |
| $t_\infty$ | 0.25 | 0.75 | 0.25 | 0.75 | $= \vec{x}P^\infty$ |

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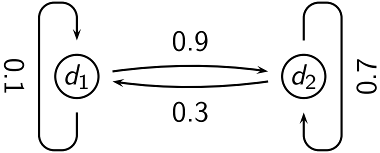
| $t_3$ | 0.252 | 0.748 | 0.2496 | 0.7504 | $= \vec{x}P^4$ |
| | | | | ... | ... |
| $t_\infty$ | 0.25 | 0.75 | 0.25 | 0.75 | $= \vec{x}P^\infty$ |

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

# Power method: Example

- What is the PageRank / steady state in this example?




- The steady state distribution (= the PageRanks) in this example are 0.25 for $d_1$ and 0.75 for $d_2$. □

# Exercise: Compute PageRank using power method

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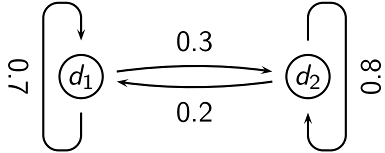

# Solution

# Solution

| | $x_1$ $P_t(d_1)$ | $x_2$ $P_t(d_2)$ | |
|---|---|---|---|
| | | | $P_{11} = 0.7$ $P_{12} = 0.3$ <br> $P_{21} = 0.2$ $P_{22} = 0.8$ |
| $t_0$ | 0 | 1 | |
| $t_1$ | | | |
| $t_2$ | | | |
| $t_3$ | | | |
| $t_\infty$ | | | |

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
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|---|---|---|---|---|
| | | | $P_{11} = 0.7$ | $P_{12} = 0.3$ |
| | | | $P_{21} = 0.2$ | $P_{22} = 0.8$ |
| $t_0$ | 0 | 1 | 0.2 | 0.8 |
| $t_1$ | | | | |
| $t_2$ | | | | |
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| $t_2$ | | | | |
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| $t_0$ | 0 | 1 | 0.2 | 0.8 |
| $t_1$ | 0.2 | 0.8 | 0.3 | 0.7 |
| $t_2$ | 0.3 | 0.7 | 0.35 | 0.65 |
| $t_3$ | | | | |
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| $t_2$ | 0.3 | 0.7 | 0.35 | 0.65 |
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| $t_3$ | 0.35 | 0.65 | 0.375 | 0.625 |
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| | | | | ... |
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  - Retrieve pages satisfying the query
  - Rank them by their PageRank
  - Return reranked list to the user

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- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query [video service]
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both *video* and *service*.
  - If we rank all Boolean hits according to PageRank, then the Yahoo home page would be top-ranked.

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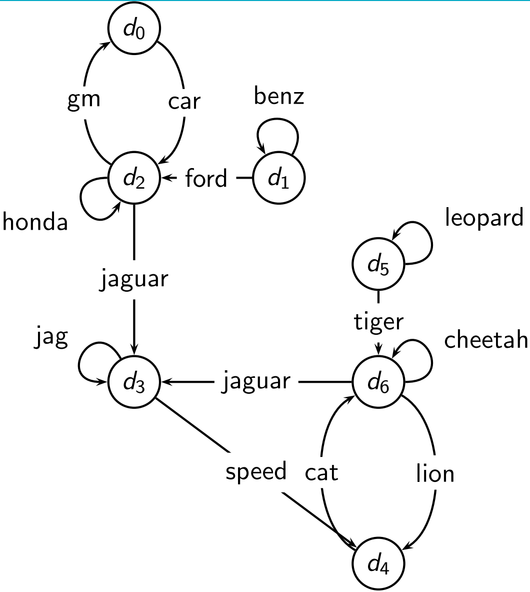
- $\rightarrow$ see lecture on Learning to Rank

# Example web graph



# Transition (probability) matrix

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| | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $d_1$ | 0.00 | 0.50 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 |
| $d_2$ | 0.33 | 0.00 | 0.33 | 0.33 | 0.00 | 0.00 | 0.00 |
| $d_3$ | 0.00 | 0.00 | 0.00 | 0.50 | 0.50 | 0.00 | 0.00 |
| $d_4$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| $d_5$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.50 |
| $d_6$ | 0.00 | 0.00 | 0.00 | 0.33 | 0.33 | 0.00 | 0.33 |

# Transition matrix with teleporting

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| | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0.02 | 0.02 | 0.88 | 0.02 | 0.02 | 0.02 | 0.02 |
| $d_1$ | 0.02 | 0.45 | 0.45 | 0.02 | 0.02 | 0.02 | 0.02 |
| $d_2$ | 0.31 | 0.02 | 0.31 | 0.31 | 0.02 | 0.02 | 0.02 |
| $d_3$ | 0.02 | 0.02 | 0.02 | 0.45 | 0.45 | 0.02 | 0.02 |
| $d_4$ | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.88 |
| $d_5$ | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 | 0.45 | 0.45 |
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# Power method vectors $\vec{x}P^k$

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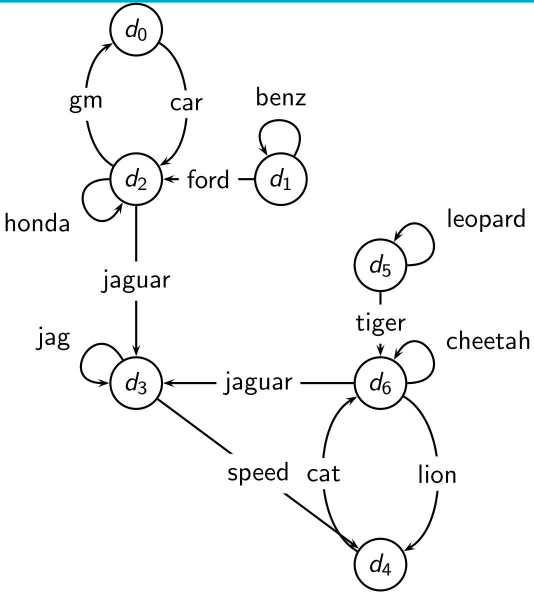
| | $\vec{x}$ | $\vec{x}P^1$ | $\vec{x}P^2$ | $\vec{x}P^3$ | $\vec{x}P^4$ | $\vec{x}P^5$ | $\vec{x}P^6$ | $\vec{x}P^7$ | $\vec{x}P^8$ | $\vec{x}P^9$ | $\vec{x}P^{10}$ | $\vec{x}P^{11}$ | $\vec{x}P^{12}$ | $\vec{x}P^{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_0$ | 0.14 | 0.06 | 0.09 | 0.07 | 0.07 | 0.06 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| $d_1$ | 0.14 | 0.08 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| $d_2$ | 0.14 | 0.25 | 0.18 | 0.17 | 0.15 | 0.14 | 0.13 | 0.12 | 0.12 | 0.12 | 0.12 | 0.11 | 0.11 | 0.11 |
| $d_3$ | 0.14 | 0.16 | 0.23 | 0.24 | 0.24 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| $d_4$ | 0.14 | 0.12 | 0.16 | 0.19 | 0.19 | 0.20 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 |
| $d_5$ | 0.14 | 0.08 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| $d_6$ | 0.14 | 0.25 | 0.23 | 0.25 | 0.27 | 0.28 | 0.29 | 0.29 | 0.30 | 0.30 | 0.30 | 0.30 | 0.31 | 0.31 |

# Example web graph




| | PageRank |
|---|---|
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| $d_1$ | 0.04 |
| $d_2$ | 0.11 |
| $d_3$ | 0.25 |
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PageRank(d2)< PageRank(d6): why?

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  - However, variants of a page's PageRank are still an essential part of ranking.
  - Adressing link spam is difficult and crucial. □

# Outline

1. Recap
2. Anchor text
3. Citation analysis
4. PageRank
5. HITS: Hubs & Authorities

# HITS – Hyperlink-Induced Topic Search

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  - By definition: Links to authority pages occur repeatedly on hub pages.
- Most approaches to search (including PageRank ranking) don't make the distinction between these two very different types of relevance. □

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- Circular definition – we will turn this into an iterative computation.

# Example for hubs and authorities

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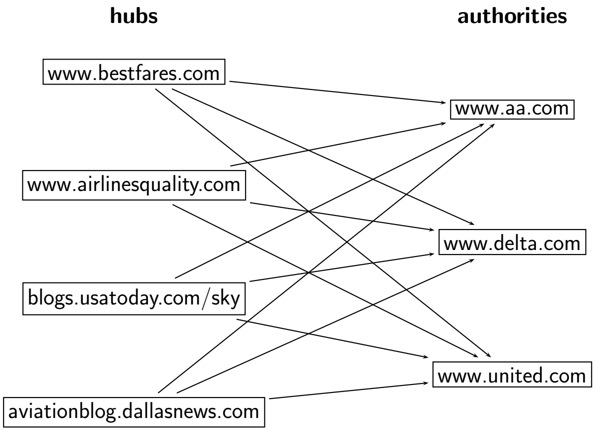

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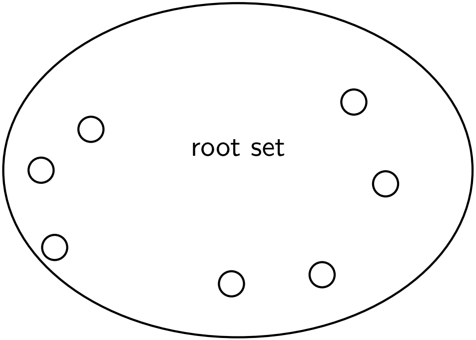
- Call this larger set the base set
- Finally, compute hubs and authorities for the base set (which we'll view as a small web graph)

# Root set and base set (1)

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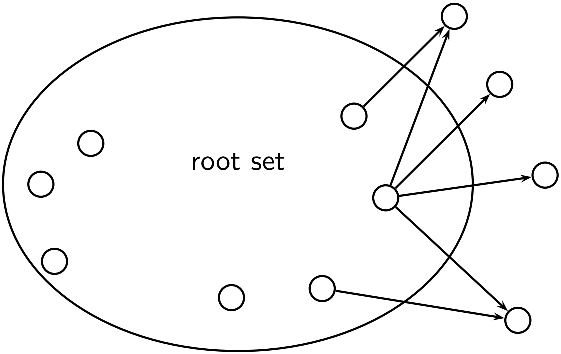


Nodes that root set nodes link to

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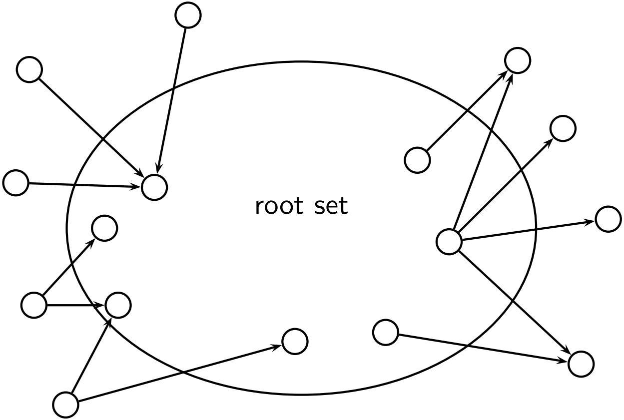


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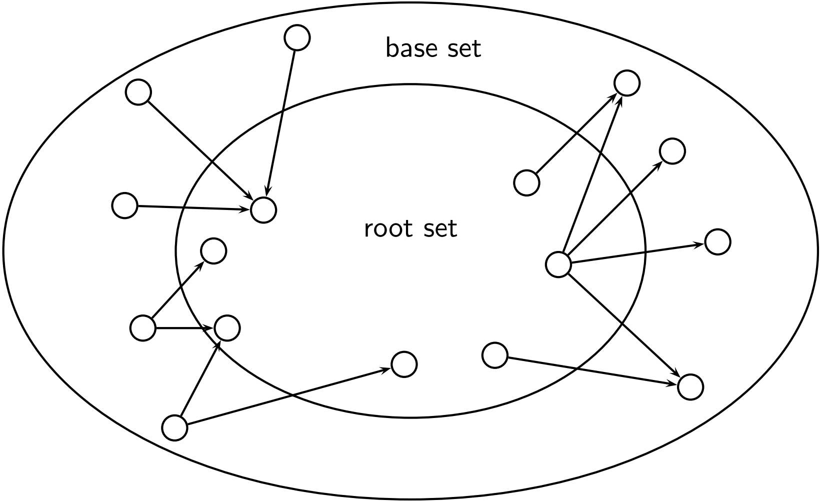


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  - Output pages with highest $a$ scores as top authorities
  - So we output two ranked lists

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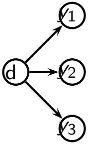

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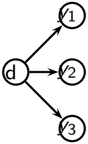
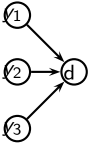



- For all $d$: $a(d) = \sum_{y \mapsto d} h(y)$



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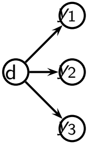


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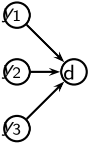


- Iterate these two steps until convergence

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  - Scaling factor doesn't really matter.
  - We care about the relative (as opposed to absolute) values of the scores.
- In most cases, the algorithm converges after a few iterations.

# Authorities for query [Chicago Bulls]

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| | |
|---|---|
| 0.85 | www.nba.com/bulls |
| 0.25 | www.essex1.com/people/jmiller/bulls.htm<br>"da Bulls" |
| 0.20 | www.nando.net/SportServer/basketball/nba/chi.html<br>"The Chicago Bulls" |
| 0.15 | users.aol.com/rynocub/bulls.htm<br>"The Chicago Bulls Home Page" |
| 0.13 | www.geocities.com/Colosseum/6095<br>"Chicago Bulls" |

(Ben-Shaul et al, WWW8)

# *The* authority page for [Chicago Bulls]

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# Hubs for query [Chicago Bulls]

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| | |
|---|---|
| 1.62 | www.geocities.com/Colosseum/1778<br>"Unbelieveabulls!!!!!" |
| 1.24 | www.webring.org/cgi-bin/webring?ring=chbulls<br>"Erin's Chicago Bulls Page" |
| 0.74 | www.geocities.com/Hollywood/Lot/3330/Bulls.html<br>"Chicago Bulls" |
| 0.52 | www.nobull.net/web_position/kw-search-15-M2.htm<br>"Excite Search Results: bulls" |
| 0.52 | www.halcyon.com/wordsltd/bball/bulls.htm<br>"Chicago Bulls Links" |

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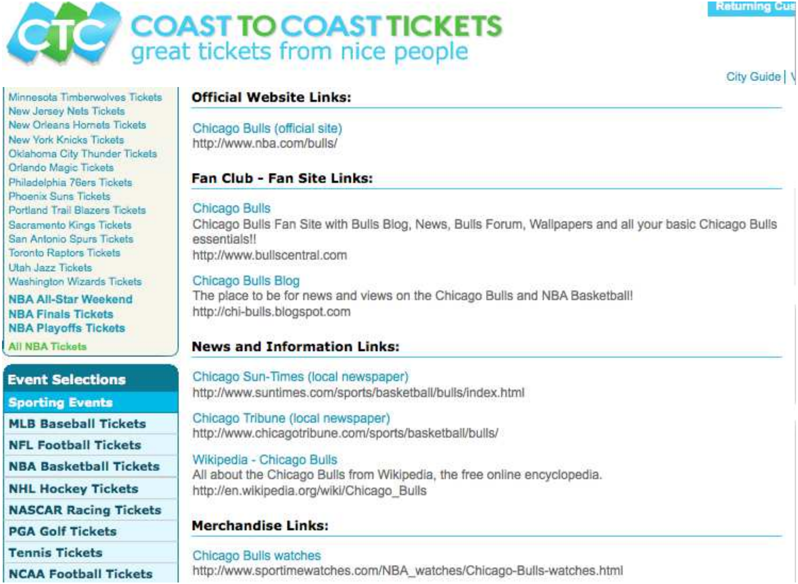

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- In theory, an English query can retrieve Japanese-language pages!
  - If supported by the link structure between English and Japanese pages
- Danger: topic drift – the pages found by following links may not be related to the original query.

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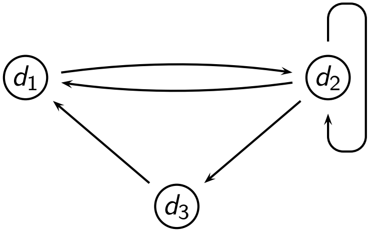

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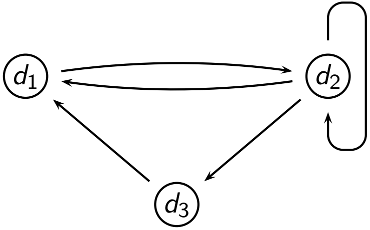


| | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|
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  - Iterate until convergence □

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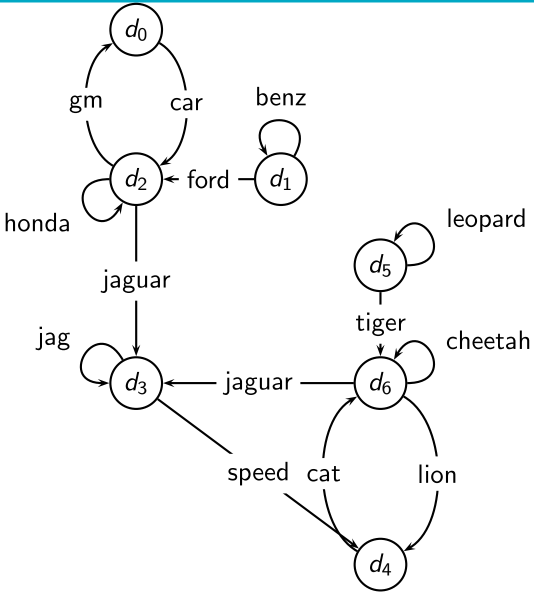
- Thus, $\vec{h}$ is an eigenvector of $AA^T$ and $\vec{a}$ is an eigenvector of $A^TA$.
- So the HITS algorithm is actually a special case of the power method and hub and authority scores are eigenvector values.
- HITS and PageRank both formalize link analysis as eigenvector problems.

# Example web graph



# Raw matrix $A$ for HITS

| | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ | $d_6$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $d_1$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| $d_2$ | 1 | 0 | 1 | 2 | 0 | 0 | 0 |
| $d_3$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $d_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $d_5$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $d_6$ | 0 | 0 | 0 | 2 | 1 | 0 | 1 |

# Hub vectors $h_0, \vec{h}_i = \frac{1}{d_i} A \cdot \vec{a}_i, i \geq 1$

| | $\vec{h}_0$ | $\vec{h}_1$ | $\vec{h}_2$ | $\vec{h}_3$ | $\vec{h}_4$ | $\vec{h}_5$ |
|---|---|---|---|---|---|---|
| $d_0$ | 0.14 | 0.06 | 0.04 | 0.04 | 0.03 | 0.03 |
| $d_1$ | 0.14 | 0.08 | 0.05 | 0.04 | 0.04 | 0.04 |
| $d_2$ | 0.14 | 0.28 | 0.32 | 0.33 | 0.33 | 0.33 |
| $d_3$ | 0.14 | 0.14 | 0.17 | 0.18 | 0.18 | 0.18 |
| $d_4$ | 0.14 | 0.06 | 0.04 | 0.04 | 0.04 | 0.04 |
| $d_5$ | 0.14 | 0.08 | 0.05 | 0.04 | 0.04 | 0.04 |
| $d_6$ | 0.14 | 0.30 | 0.33 | 0.34 | 0.35 | 0.35 |

# Authority vectors $\vec{a}_i = \frac{1}{c_i} A^T \cdot \vec{h}_{i-1}, i \geq 1$

| | $\vec{a}_1$ | $\vec{a}_2$ | $\vec{a}_3$ | $\vec{a}_4$ | $\vec{a}_5$ | $\vec{a}_6$ | $\vec{a}_7$ |
|---|---|---|---|---|---|---|---|
| $d_0$ | 0.06 | 0.09 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| $d_1$ | 0.06 | 0.03 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| $d_2$ | 0.19 | 0.14 | 0.13 | 0.12 | 0.12 | 0.12 | 0.12 |
| $d_3$ | 0.31 | 0.43 | 0.46 | 0.46 | 0.46 | 0.47 | 0.47 |
| $d_4$ | 0.13 | 0.14 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |
| $d_5$ | 0.06 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |
| $d_6$ | 0.19 | 0.14 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 |

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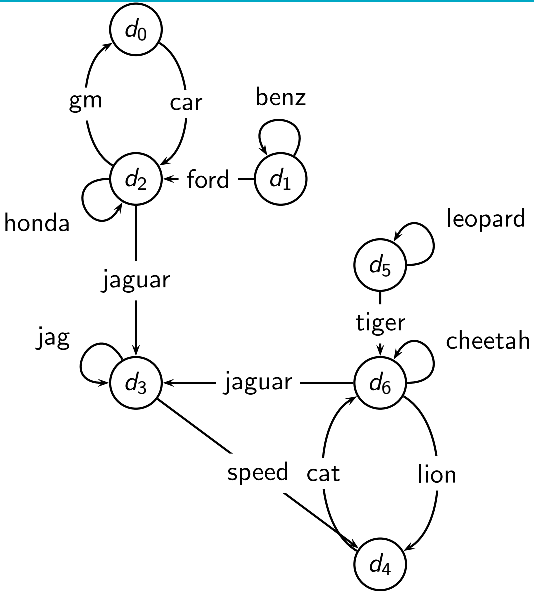


| | $a$ | $h$ |
|---|---|---|
| $d_0$ | 0.10 | 0.03 |
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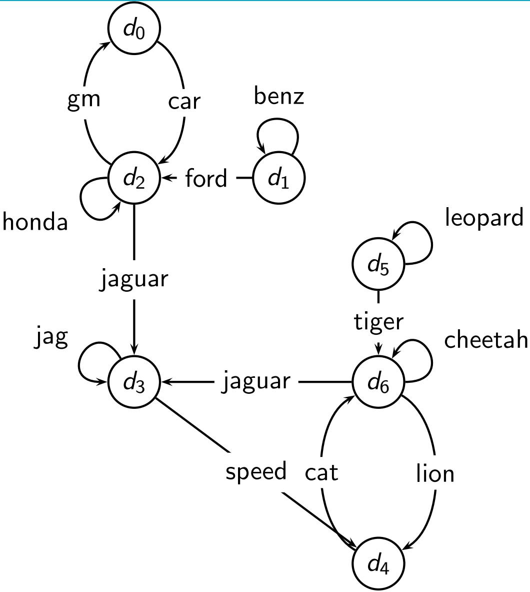


Pages with highest in-degree: $d_2$, $d_3$, $d_6$

Pages with highest out-degree: $d_2$, $d_6$

Pages with highest PageRank: $d_6$

Pages with highest hub score: $d_6$ (close: $d_2$)

Pages with highest authority score: $d_3$ □

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- These two are orthogonal.
  - We could also apply HITS to the entire web and PageRank to a small base set.
- Claim: On the web, a good hub almost always is also a good authority.
- The actual difference between PageRank ranking and HITS ranking is therefore not as large as one might expect. □

# Exercise

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- Why is a good hub almost always also a good authority? □

# Take-away today

- Anchor text: What exactly are links on the web and why are they important for IR?
- Citation analysis: the mathematical foundation of PageRank and link-based ranking
- PageRank: the original algorithm that was used for link-based ranking on the web
- Hubs & Authorities: an alternative link-based ranking algorithm

# Resources

- Chapter 21 of IIR
- Resources at `http://cislmu.org`
  - American Mathematical Society article on PageRank (popular science style)
  - Jon Kleinberg's home page (main person behind HITS)
  - A Google bomb and its defusing
  - Google's official description of PageRank: *PageRank reflects our view of the importance of web pages by considering more than 500 million variables and 2 billion terms. Pages that we believe are important pages receive a higher PageRank and are more likely to appear at the top of the search results.*