

# Introduction to Information Retrieval

<http://informationretrieval.org>

## IIR 13: Text Classification & Naive Bayes

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# Overview

- 1 Recap
- 2 Text classification
- 3 Naive Bayes
- 4 NB theory
- 5 Evaluation of TC

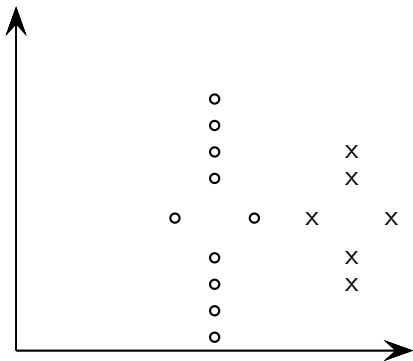
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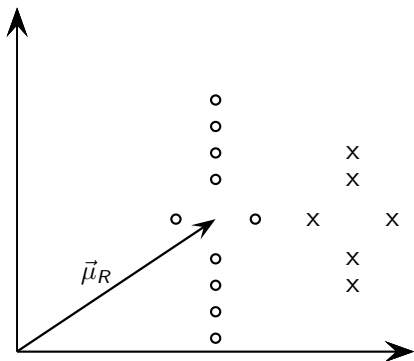
# Relevance feedback: Basic idea

- The user issues a (short, simple) query.
- The search engine returns a set of documents.
- User marks some docs as relevant, some as nonrelevant.
- Search engine computes a new representation of the information need – should be better than the initial query.
- Search engine runs new query and returns new results.
- New results have (hopefully) better recall.

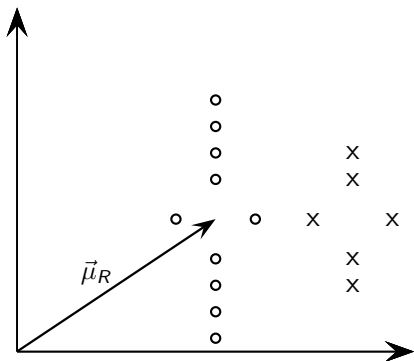
# Rocchio illustrated



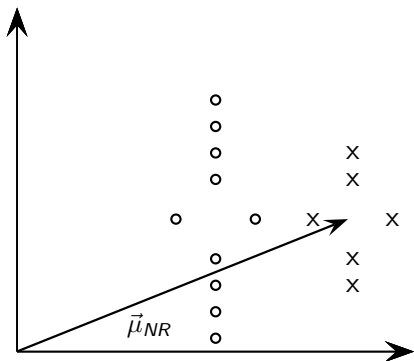
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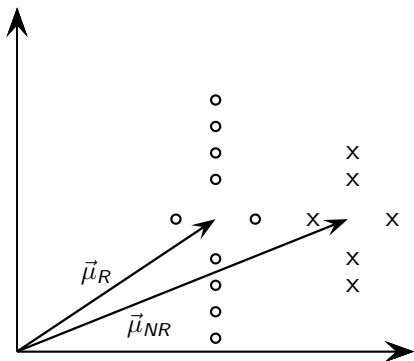


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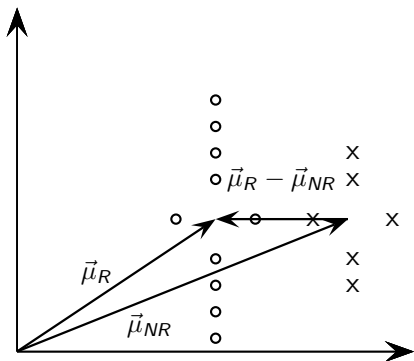




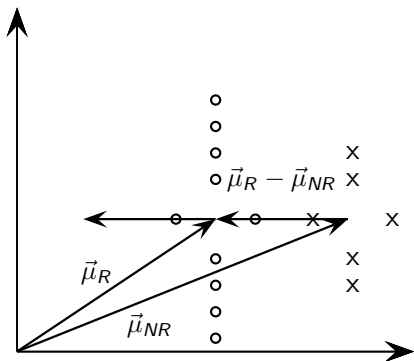
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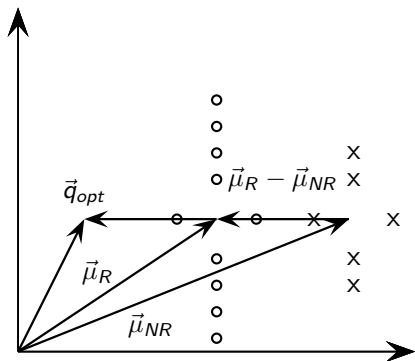
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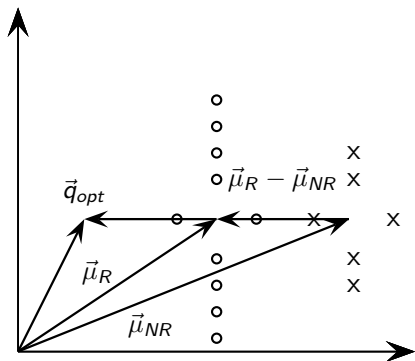
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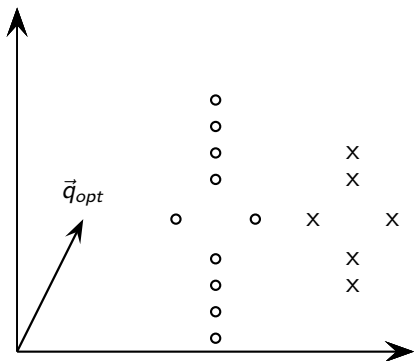
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# Types of query expansion

- Manual thesaurus (maintained by editors, e.g., PubMed)
- Automatically derived thesaurus (e.g., based on co-occurrence statistics)
- Query-equivalence based on query log mining (common on the web as in the “palm” example)

# Query expansion at search engines

- Main source of query expansion at search engines: query logs
- Example 1: After issuing the query [herbs], users frequently search for [herbal remedies].
  - → “herbal remedies” is potential expansion of “herb”.
- Example 2: Users searching for [flower pix] frequently click on the URL [photobucket.com/flower](http://photobucket.com/flower). Users searching for [flower clipart] frequently click on the [same URL](#).
  - → “flower clipart” and “flower pix” are potential expansions of each other.



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- Naive Bayes: simple baseline text classifier
- Theory: derivation of Naive Bayes classification rule & analysis
- Evaluation of text classification: how do we know it worked / didn't work?

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# A text classification task: Email spam filtering

From: '' <takworldd@hotmail.com>

Subject: real estate is the only way... gem oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

=====  
Click Below to order:

<http://www.wholesaledaily.com/sales/nmd.htm>

=====

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Using a learning method or **learning algorithm**, we then wish to learn a **classifier**  $\gamma$  that maps documents to classes:

$$\gamma : \mathbb{X} \rightarrow \mathbb{C}$$

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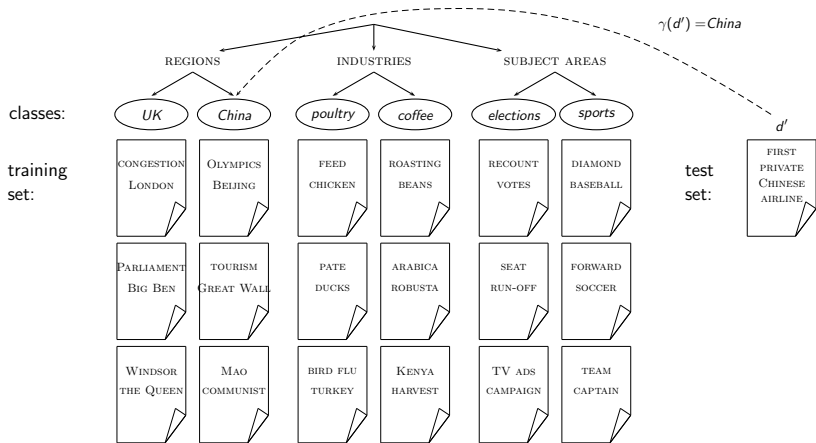
# Formal definition of TC: Application/Testing

Given: a description  $d \in \mathbb{X}$  of a document

Determine:  $\gamma(d) \in \mathbb{C}$ , that is, the class that is most appropriate for  $d$



# Topic classification



# Exercise

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- Find examples of uses of text classification in information retrieval

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- The automatic detection of spam pages (spam vs. nonspam)
- Sentiment detection: is a movie or product review positive or negative (positive vs. negative)
- Topic-specific or *vertical* search – restrict search to a “vertical” like “related to health” (relevant to vertical vs. not)



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- Scaling manual classification is difficult and expensive.
- → We need automatic methods for classification.

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- Building and maintaining rule-based classification systems is cumbersome and expensive.

# A Verity topic (a complex classification rule)

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```

comment line      # Beginning of art topic definition
top-level topic  art ACCRUE
                 /author = "fsmith"
topic definition modifiers } /date = "30-Dec-01"
                           /annotation = "Topic created
                             by fsmith"
subtopic         * 0.70 film ACCRUE
                 ** 0.50 STEM
                 /wordtext = film
evidencetopic   ** 0.50 WORD
topic definition modifier } /wordtext = ballet
evidencetopic     ** 0.50 STEM
topic definition modifier } /wordtext = dance
evidencetopic     ** 0.50 WORD
topic definition modifier } /wordtext = opera
evidencetopic     ** 0.30 WORD
topic definition modifier } /wordtext = symphony
subtopic         * 0.70 visual-arts ACCRUE
                 ** 0.50 WORD
                 /wordtext = painting
                 ** 0.50 WORD
                 /wordtext = sculpture
subtopic         * 0.70 film ACCRUE
                 ** 0.50 STEM
                 /wordtext = film
subtopic         ** 0.50 motion-picture PHRAS
                 *** 1.00 WORD
                 /wordtext = motion
                 *** 1.00 WORD
                 /wordtext = picture
                 ** 0.50 STEM
                 /wordtext = movie
subtopic         * 0.50 video ACCRUE
                 ** 0.50 STEM
                 /wordtext = video
                 ** 0.50 STEM
                 /wordtext = vcr
                 # End of art topic

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- But this manual classification can be done by non-experts.

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- If a document's terms do not provide clear evidence for one class vs. another, we choose the  $c$  with highest  $P(c)$ .

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- The best class is the most likely or **maximum a posteriori (MAP) class**  $c_{\text{map}}$ :

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} \hat{P}(c|d) = \arg \max_{c \in \mathbb{C}} \hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)$$

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- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k | c)]$$

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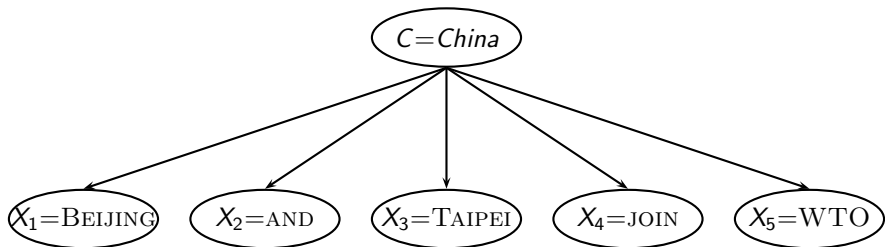
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- $T_{ct}$  is the number of tokens of  $t$  in training documents from class  $c$  (includes multiple occurrences)
- We've made a **Naive Bayes independence assumption** here:  
 $\hat{P}(t_k|c) = \hat{P}(t_k|c)$ , independent of position

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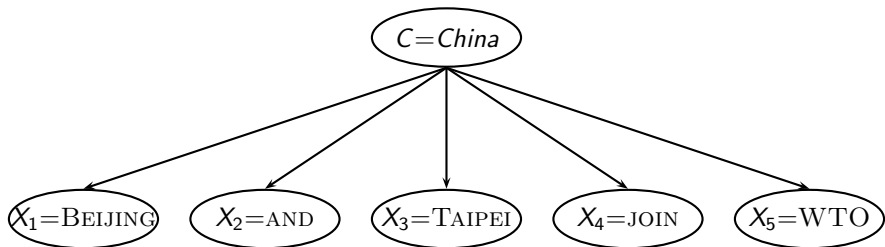


$$P(\text{China}|d) \propto P(\text{China}) \cdot P(\text{BEIJING}|\text{China}) \cdot P(\text{AND}|\text{China}) \\ \cdot P(\text{TAIPEI}|\text{China}) \cdot P(\text{JOIN}|\text{China}) \cdot P(\text{WTO}|\text{China})$$

- If WTO never occurs in class China in the train set:

$$\hat{P}(\text{WTO}|\text{China}) = \frac{T_{\text{China},\text{WTO}}}{\sum_{t' \in V} T_{\text{China},t'}} = \frac{0}{\sum_{t' \in V} T_{\text{China},t'}} = 0$$

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# The problem with maximum likelihood estimates: Zeros (cont)

- If there are no occurrences of *WTO* in documents in class *China*, we get a zero estimate:

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$$\hat{P}(\text{WTO}|\text{China}) = \frac{T_{\text{China},\text{WTO}}}{\sum_{t' \in V} T_{\text{China},t'}} = 0$$

- $\rightarrow$  We will get  $P(\text{China}|d) = 0$  for any document that contains *WTO*!

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# To avoid zeros: Add-one smoothing

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- Assign the document to the class with the largest score

# Naive Bayes: Training

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TRAINMULTINOMIALNB( $\mathbb{C}, \mathbb{D}$ )

```
1   $V \leftarrow \text{EXTRACTVOCABULARY}(\mathbb{D})$ 
2   $N \leftarrow \text{COUNTDOCS}(\mathbb{D})$ 
3  for each  $c \in \mathbb{C}$ 
4  do  $N_c \leftarrow \text{COUNTDOCSINCLASS}(\mathbb{D}, c)$ 
5      $\text{prior}[c] \leftarrow N_c/N$ 
6      $\text{text}_c \leftarrow \text{CONCATENATETEXTOFALLDOCSINCLASS}(\mathbb{D}, c)$ 
7     for each  $t \in V$ 
8     do  $T_{ct} \leftarrow \text{COUNTTOKENSOFTERM}(\text{text}_c, t)$ 
9     for each  $t \in V$ 
10    do  $\text{condprob}[t][c] \leftarrow \frac{T_{ct}+1}{\sum_{t'} (T_{ct'}+1)}$ 
11 return  $V, \text{prior}, \text{condprob}$ 
```



# Naive Bayes: Testing

# Naive Bayes: Testing

APPLYMULTINOMIALNB( $\mathbb{C}$ ,  $V$ , *prior*, *condprob*,  $d$ )

1  $W \leftarrow \text{EXTRACTTOKENSFROMDOC}(V, d)$

2 **for each**  $c \in \mathbb{C}$

3 **do**  $\text{score}[c] \leftarrow \log \text{prior}[c]$

4     **for each**  $t \in W$

5         **do**  $\text{score}[c] + = \log \text{condprob}[t][c]$

6 **return**  $\arg \max_{c \in \mathbb{C}} \text{score}[c]$

# Exercise: Estimate parameters, classify test set

	docID	words in document	in $c = \textit{China}$ ?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V} (T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

( $B$  is the number of bins – in this case the number of different words or the size of the vocabulary  $|V| = M$ )

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} [\hat{P}(c) \cdot \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)]$$



# Example: Parameter estimates

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Priors:  $\hat{P}(c) = 3/4$  and  $\hat{P}(\bar{c}) = 1/4$

Conditional probabilities:

$$\hat{P}(\text{CHINESE}|c) = (5 + 1)/(8 + 6) = 6/14 = 3/7$$

$$\hat{P}(\text{TOKYO}|c) = \hat{P}(\text{JAPAN}|c) = (0 + 1)/(8 + 6) = 1/14$$

$$\hat{P}(\text{CHINESE}|\bar{c}) = (1 + 1)/(3 + 6) = 2/9$$

$$\hat{P}(\text{TOKYO}|\bar{c}) = \hat{P}(\text{JAPAN}|\bar{c}) = (1 + 1)/(3 + 6) = 2/9$$

The denominators are  $(8 + 6)$  and  $(3 + 6)$  because the lengths of  $\text{text}_c$  and  $\text{text}_{\bar{c}}$  are 8 and 3, respectively, and because the constant  $B$  is 6 as the vocabulary consists of six terms.

# Example: Classification

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$$\hat{P}(c|d_5) \propto 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003$$

$$\hat{P}(\bar{c}|d_5) \propto 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001$$

Thus, the classifier assigns the test document to  $c = \textit{China}$ . The reason for this classification decision is that the three occurrences of the positive indicator `CHINESE` in  $d_5$  outweigh the occurrences of the two negative indicators `JAPAN` and `TOKYO`.



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mode	time complexity
training	$\Theta( \mathbb{D} L_{\text{ave}} +  \mathbb{C}  V )$
testing	$\Theta(L_a +  \mathbb{C} M_a) = \Theta( \mathbb{C} M_a)$

- $L_{\text{ave}}$ : average length of a training doc,  $L_a$ : length of the test doc,  $M_a$ : number of distinct terms in the test doc,  $\mathbb{D}$ : training set,  $V$ : vocabulary,  $\mathbb{C}$ : set of classes

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- Generally:  $|\mathbb{C}||V| < |\mathbb{D}|L_{\text{ave}}$
- Test time is also linear (in the length of the test document).
- Thus: **Naive Bayes is linear** in the size of the training set (training) and the test document (testing). This is **optimal**.

# Outline

- 1 Recap
- 2 Text classification
- 3 Naive Bayes
- 4 NB theory
- 5 Evaluation of TC



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- We will formally derive the classification rule ...
- ... and make our assumptions explicit.

# Derivation of Naive Bayes rule

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We want to find the class that is most likely given the document:

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} P(c|d)$$

Apply Bayes rule  $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ :

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} \frac{P(d|c)P(c)}{P(d)}$$

Drop denominator since  $P(d)$  is the same for all classes:

$$c_{\text{map}} = \arg \max_{c \in \mathbb{C}} P(d|c)P(c)$$

# Too many parameters / sparseness

$$\begin{aligned}c_{\text{map}} &= \arg \max_{c \in \mathbb{C}} P(d|c)P(c) \\ &= \arg \max_{c \in \mathbb{C}} P(\langle t_1, \dots, t_k, \dots, t_{n_d} \rangle | c)P(c)\end{aligned}$$

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- This is the problem of **data sparseness**.

# Naive Bayes conditional independence assumption

To reduce the number of parameters to a manageable size, we make the **Naive Bayes conditional independence assumption**:

$$P(d|c) = P(\langle t_1, \dots, t_{n_d} \rangle | c) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | c)$$

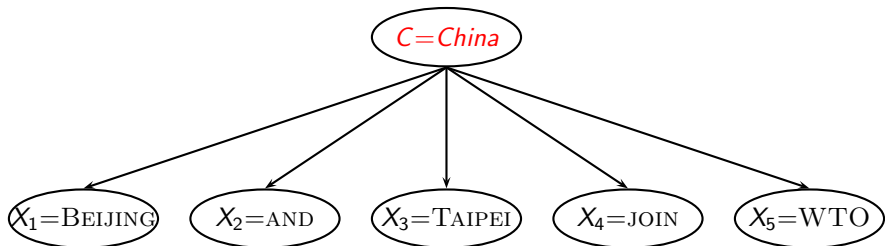
We assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities  $P(X_k = t_k | c)$ .

Recall from earlier the estimates for these conditional probabilities:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

# Generative model

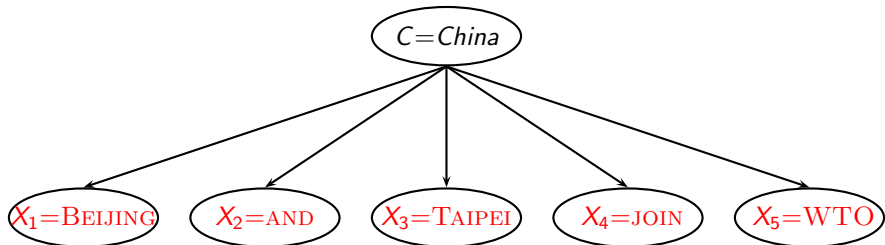
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$$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$$

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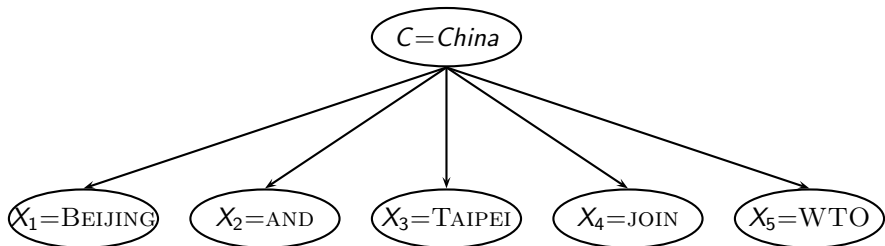
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- Generate a class with probability  $P(c)$
- Generate each of the words (in their respective positions), conditional on the class, but independent of each other, with probability  $P(t_k|c)$
- To classify docs, we “reengineer” this process and find the class that is most likely to have generated the doc.

# Second independence assumption



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- $\hat{P}(X_{k_1} = t|c) = \hat{P}(X_{k_2} = t|c)$

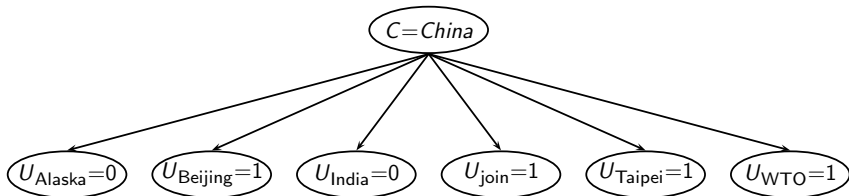
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- For example, for a document in the class *UK*, the probability of generating QUEEN in the first position of the document is the same as generating it in the last position.
- The two independence assumptions amount to the **bag of words** model.

# A different Naive Bayes model: Bernoulli model



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- Exercise
  - Examples for why conditional independence assumption is not really true?
  - Examples for why positional independence assumption is not really true?
- How can Naive Bayes work if it makes such inappropriate assumptions?

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true probability $P(c d)$	0.6	0.4	$c_1$
$\hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k c)$	0.00099	0.00001	
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- ... but it often performs well at accurate prediction (choosing the correct class).

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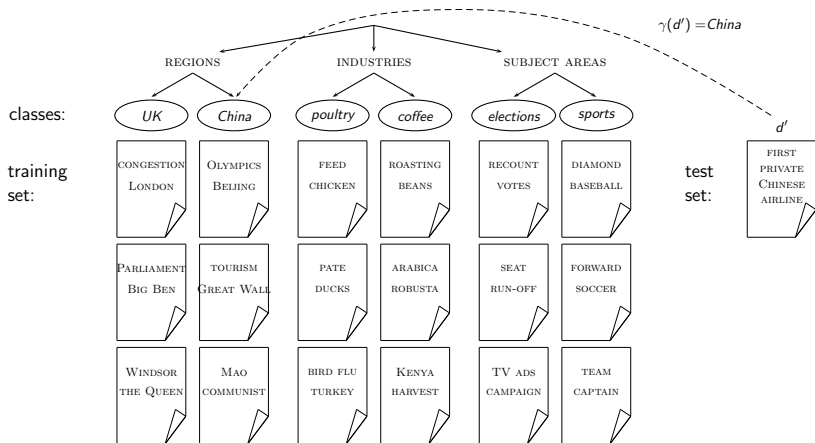
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# Evaluation on Reuters



# Example: The Reuters collection

symbol	statistic	value
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type of class	number	examples
region	366	UK, China
industry	870	poultry, coffee
subject area	126	elections, sports



# A Reuters document

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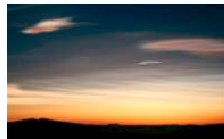
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## Extreme conditions create rare Antarctic clouds

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SYDNEY (Reuters) - Rare, mother-of-pearl colored clouds caused by extreme weather conditions above Antarctica are a possible indication of global warming, Australian scientists said on Tuesday.

Known as nacreous clouds, the spectacular formations showing delicate wisps of colors were photographed in the sky over an Australian

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- Evaluation must be done on test data that are independent of the training data, i.e., training and test sets are disjoint.
- It's easy to get good performance on a test set that was available to the learner during training (e.g., just memorize the test set).
- Measures: Precision, recall,  $F_1$ , classification accuracy

# Precision $P$ and recall $R$

	in the class	not in the class
predicted to be in the class	true positives (TP)	false positives (FP)
predicted to not be in the class	false negatives (FN)	true negatives (TN)

TP, FP, FN, TN are counts of documents. The sum of these four counts is the total number of documents.

$$\text{precision: } P = TP / (TP + FP)$$

$$\text{recall: } R = TP / (TP + FN)$$

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- This is the **harmonic mean** of  $P$  and  $R$ :  $\frac{1}{F} = \frac{1}{2} \left( \frac{1}{P} + \frac{1}{R} \right)$

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  - Compute  $F_1$  for aggregate TP, FP, FN

# $F_1$ scores for Naive Bayes vs. other methods

(a)	NB	Rocchio	kNN	SVM	
micro-avg-L (90 classes)	80	85	86	89	
macro-avg (90 classes)	47	59	60	60	

(b)	NB	Rocchio	kNN	trees	SVM
earn	96	93	97	98	98
acq	88	65	92	90	94
money-fx	57	47	78	66	75
grain	79	68	82	85	95
crude	80	70	86	85	89
trade	64	65	77	73	76
interest	65	63	74	67	78
ship	85	49	79	74	86
wheat	70	69	77	93	92
corn	65	48	78	92	90
micro-avg (top 10)	82	65	82	88	92
micro-avg-D (118 classes)	75	62	n/a	n/a	87

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Naive Bayes does pretty well, but some methods beat it consistently (e.g., SVM).

# Take-away today

- Text classification: definition & relevance to information retrieval
- Naive Bayes: simple baseline text classifier
- Theory: derivation of Naive Bayes classification rule & analysis
- Evaluation of text classification: how do we know it worked / didn't work?

# Resources

- Chapter 13 of IIR
- Resources at <http://cis1mu.org>
  - Weka: A data mining software package that includes an implementation of Naive Bayes
  - Reuters-21578 – text classification evaluation set
  - Vulgarity classifier fail