Introduction to Information Retrieval

http://informationretrieval.org

IIR 18: Latent Semantic Indexing

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Overview

Outline

Indexing anchor text

- Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than the text on the page.
- A Google bomb is a search with "bad" results due to maliciously manipulated anchor text.
  - [dangerous cult] on Google, Bing, Yahoo □

PageRank

- Model: a web surfer doing a random walk on the web
- Formalization: Markov chain
- PageRank is the long-term visit rate of the random surfer or the steady-state distribution.
- Need teleportation to ensure well-defined PageRank
- Power method to compute PageRank
  - PageRank is the principal left eigenvector of the transition probability matrix.

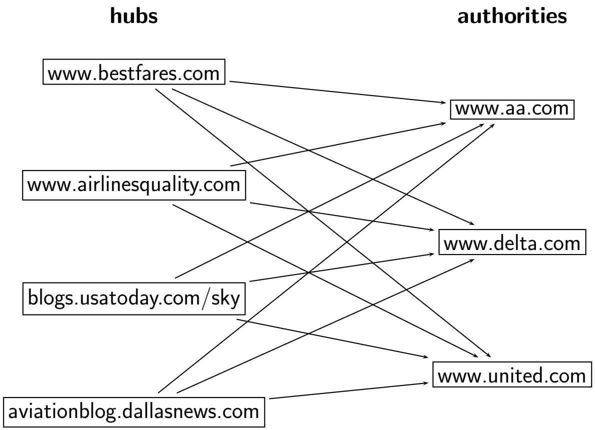
Computing PageRank: Power method

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
				...	
t_∞	0.25	0.75	0.25	0.75	$= \vec{x}P^\infty$

PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

HITS: Hubs and authorities



HITS update rules

- A: link matrix
- $\vec{h}$: vector of hub scores
- $\vec{a}$: vector of authority scores
- HITS algorithm:
  - Compute $\vec{h} = A\vec{a}$
  - Compute $\vec{a} = A^T\vec{h}$
  - Iterate until convergence
  - Output (i) list of hubs ranked according to hub score and (ii) list of authorities ranked according to authority score

Take-away today

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- Latent Semantic Indexing (LSI) / Singular Value Decomposition: The math

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- Latent Semantic Indexing (LSI) / Singular Value Decomposition: The math
- SVD used for dimensionality reduction
- LSI: SVD in information retrieval
- LSI as clustering

Outline

Recall: Term-document matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
anthony	5.25	3.18	0.0	0.0	0.0	0.35
brutus	1.21	6.10	0.0	1.0	0.0	0.0
caesar	8.59	2.54	0.0	1.51	0.25	0.0
calpurnia	0.0	1.54	0.0	0.0	0.0	0.0
cleopatra	2.85	0.0	0.0	0.0	0.0	0.0
mercy	1.51	0.0	1.90	0.12	5.25	0.88
worser	1.37	0.0	0.11	4.15	0.25	1.95
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Today: Can we transform this matrix, so that we get a better measure of similarity between documents and queries?

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- We will then use the SVD to compute a new, improved term-document matrix C'.
- We'll get better similarity values out of C' (compared to C).
- Using SVD for this purpose is called latent semantic indexing or LSI.

Example of $C = U\Sigma V^T$: The matrix C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

This is a standard term-document matrix.

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This is a standard term-document matrix.

Actually, we use a non-weighted matrix here to simplify the example.

Example of $C = U\Sigma V^T$: The matrix U

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
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2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
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We'll make use of this by omitting unimportant dimensions. □

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V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
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Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

$=$

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LSI is decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.

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- The singular value matrix Σ – diagonal matrix with singular values, reflecting importance of each dimension
- Next: Why are we doing this? □

Exercise

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
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Verify that the first document has unit length.

Verify that the first two documents are orthogonal.

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$0.75^2 + 0.29^2 + 0.28^2 + 0.00^2 + 0.53^2 = 1.0059$

$-0.75 * -0.28 + -0.29 * -0.53 + 0.28 * -0.75 + 0.00 * 0.00 +$
$-0.53 * 0.29 = 0$

Outline

How we use the SVD in LSI

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  - make things dissimilar that should be similar – again, the reduced LSI representation is a better representation because it represents similarity better.

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- Analogy for “fewer details is better”
  - Image of a blue flower

How we use the SVD in LSI

- Key property: Each singular value tells us how important its dimension is.
- By setting less important dimensions to zero, we keep the important information, but get rid of the "details".
- These details may
  - be noise – in that case, reduced LSI is a better representation because it is less noisy.
  - make things dissimilar that should be similar – again, the reduced LSI representation is a better representation because it represents similarity better.
- Analogy for "fewer details is better"
  - Image of a blue flower
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  - Image of a blue flower
  - Image of a yellow flower
  - Omitting color makes is easier to see the similarity

Reducing the dimensionality to 2

U	1	2	3	4	5
ship	−0.44	−0.30	0.00	0.00	0.00
boat	−0.13	−0.33	0.00	0.00	0.00
ocean	−0.48	−0.51	0.00	0.00	0.00
wood	−0.70	0.35	0.00	0.00	0.00
tree	−0.26	0.65	0.00	0.00	0.00

Σ_2	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.00	0.00	0.00	0.00	0.00	0.00
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Actually, we only zero out singular values in Σ. This has the effect of setting the corresponding dimensions in U and V^T to zero when computing the product $C = U\Sigma V^T$. □

Reducing the dimensionality to 2

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
boat	0.36	0.36	0.16	−0.20	−0.02	−0.18
ocean	1.01	0.72	0.36	−0.04	0.16	−0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	−0.39	−0.08	0.90	0.41	0.49

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
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Recall unreduced decomposition $C = U\Sigma V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

$=$

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Original matrix C vs. reduced $C_2 = U\Sigma_2 V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	−0.08
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We can view C_2 as a two-dimensional representation of the matrix C. We have performed a dimensionality reduction to two dimensions.

Exercise

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

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Compute the similarity between d_2 and d_3 for the original matrix and for the reduced matrix.

Why the reduced matrix C_2 is better than C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

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Similarity of d_2 and d_3 in the original space: 0.

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Similarity of d_2 and d_3 in the original space: 0.

Similarity of d_2 and d_3 in the reduced space: $0.52 * 0.28 + 0.36 * 0.16 + 0.72 * 0.36 + 0.12 * 0.20 + -0.39 * -0.08 \approx 0.52$

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"boat" and "ship" are semantically similar. The "reduced" similarity measure reflects this.

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"boat" and "ship" are semantically similar. The "reduced" similarity measure reflects this.

What property of the SVD reduction is responsible for improved similarity?

Exercise: Compute matrix product

???????=

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ocean	0.15	0.27	0.10	-0.32	-0.11	-0.21
wood	-0.10	-0.19	-0.07	0.22	0.08	0.14
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Outline

1. Recap
2. Latent semantic indexing
3. Dimensionality reduction
4. LSI in information retrieval
5. Clustering

Why we use LSI in information retrieval

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- LSI takes documents that are semantically similar (= talk about the same topics), . . .
- . . . but are not similar in the vector space (because they use different words) . . .
- . . . and re-represents them in a reduced vector space . . .
- . . . in which they have higher similarity.

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- . . . in which they have higher similarity.
- Thus, LSI addresses the problems of synonymy and semantic relatedness.
- Standard vector space: Synonyms contribute nothing to document similarity.
- Desired effect of LSI: Synonyms contribute strongly to document similarity.

How LSI addresses synonymy and semantic relatedness

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- The "cost" of mapping synonyms to the same dimension is much less than the cost of collapsing unrelated words.
- SVD selects the "least costly" mapping (see below).
- Thus, it will map synonyms to the same dimension.
- But it will avoid doing that for unrelated words. □

LSI: Comparison to other approaches

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- Recap: Relevance feedback and query expansion are used to increase recall in information retrieval – if query and documents have no terms in common.

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- . . . and it has the same problems.

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- Exercise: What is the fundamental problem with this approach?

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- So LSI uses the "best possible" matrix.
- There is only one best possible matrix – unique solution (modulo signs).
- Caveat: There is only a tenuous relationship between the Frobenius norm and cosine similarity between documents. □

Data for graphical illustration of LSI

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c_1	Human machine interface for lab abc computer applications
c_2	A survey of user opinion of computer system response time
c_3	The EPS user interface management system
c_4	System and human system engineering testing of EPS
c_5	Relation of user perceived response time to error measurement
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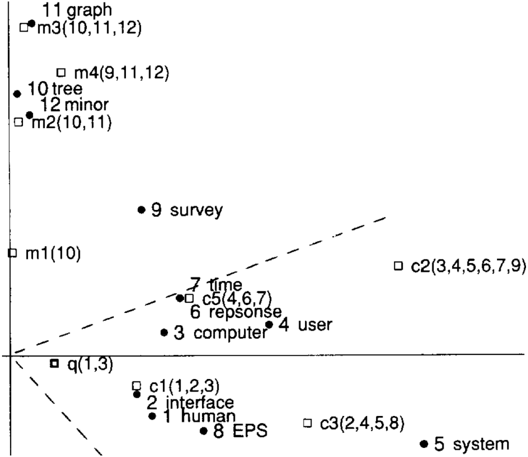
The matrix C

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

Graphical illustration of LSI: Plot of C_2



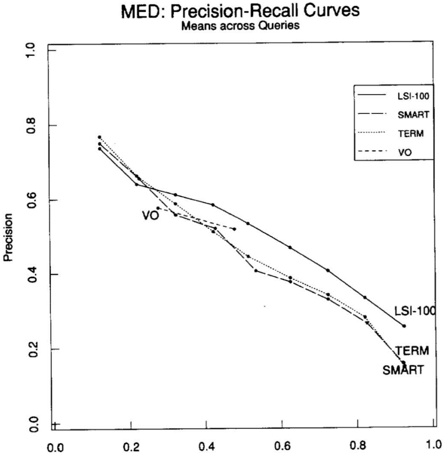
2-dimensional plot of C_2 (scaled dimensions). Circles = terms. Open squares = documents (component terms in parentheses). q = query "human computer interaction".

The dotted cone represents the region whose points are within a cosine of .9 from q . All documents about human-computer documents (c1-c5) are near q, even c3/c5 although they share no terms. None of the graph theory documents (m1-m4) are near q.

Exercise

What happens when we rank documents according to cosine similarity in the original vector space? What happens when we rank documents according to cosine similarity in the reduced vector space?

LSI performs better than vector space on MED collection



LSI-100 = LSI reduced to 100 dimensions; SMART = SMART implementation of vector space model

Outline

Exercise: Why can this be viewed as soft clustering?

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	−0.44	−0.30	0.57	0.58	0.25
boat	−0.13	−0.33	−0.59	0.00	0.73
ocean	−0.48	−0.51	−0.37	0.00	−0.61
wood	−0.70	0.35	0.15	−0.58	0.16
tree	−0.26	0.65	−0.41	0.58	−0.09

×

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

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- d_1, d_2, d_3 have negative values (most of their terms are water terms).
- d_4, d_5, d_6 have positive values (all of their terms are earth terms).

Take-away today

- Latent Semantic Indexing (LSI) / Singular Value Decomposition: The math
- SVD used for dimensionality reduction
- LSI: SVD in information retrieval
- LSI as clustering

Resources

- Chapter 18 of IIR
- Resources at `http://cislmu.org`
  - Original paper on latent semantic indexing by Deerwester et al.
  - Paper on probabilistic LSI by Thomas Hofmann
  - Word space: LSI for words